Delay-Doppler Domain Waveform Design for ISAC:

Can we go beyond the limits of the uncertainty principle?

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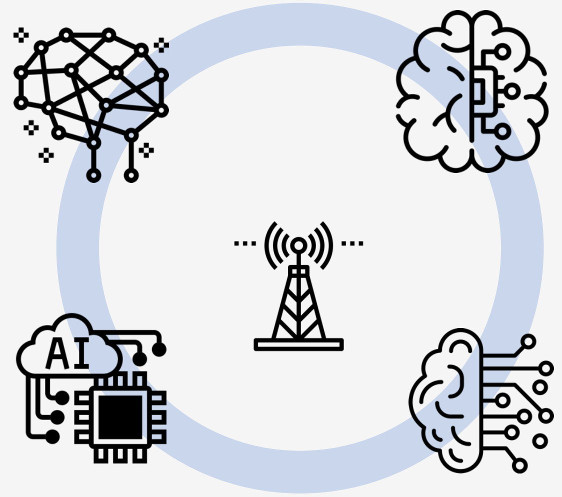
Outline

1. ISAC Waveform Requirements
2. DD Domain Waveform for ISAC
3. Ambiguity Function & Uncertainty Principle
4. Orthogonal Delay-Doppler Division Multiplexing (ODDM)
5. From the Perspective of Sequence-based Waveform Design
6. Waveform-Level Simulations
7. Conclusion

ISAC Waveform Requirements

- Connected Intelligence



Integrated Sensing and Communication (ISAC)

✓ Both communication and sensing utilize electromagnetic waves.

✓ ISAC requires waveforms that are well-suited for both functions.

- Modulation waveform：Not necessarily suitable for sensing.
- Radar waveform: Not sufficient for communication.
- A straightforward solution: Combining two types of waveforms
- Questions: How? Any underlying logic behind the combination?

A waveform is fundamentally a **continuous-time real-valued** signal.

- Modulation: Channel-oriented design, orthogonality, capacity, complexity, etc.
- Radar: High sensitivity, high time-frequency (TF) resolution

Modulation Waveform

Real-valued passband signal :

Amplitude — Frequency — Phase — Pulse (envelop) (Duration T_p, Bandwidth B_p)

Can be used for modulation

$$\sqrt{2}A_i \cos\left(2\pi(f_c + f_i(t))t + \theta_i + \theta_c\right) p_i(t)$$

Carrier frequency — Carrier phase

Complex-valued baseband signal : (coherent, ideal synchronization)

Transmit pulse (filter)

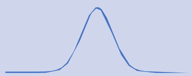
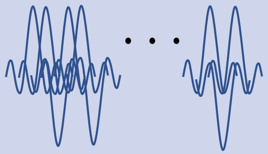
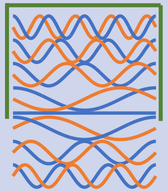
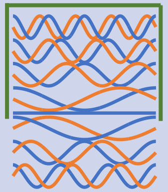
$$A_i e^{j\theta_i}\delta(t) \rightarrow \boxed{e^{j2\pi f_i(t)t}p_i(t)} \rightarrow \quad A_i e^{j\theta_i} e^{j2\pi f_i(t)t} p_i(t) = A_i e^{j\theta_i} g_i(t)$$

Orthogonal pulses / Basis functions
Core of modulation scheme

Analog modulation : $A_i, \theta_i, f_i(t)$ are continuous-valued (AM,FM,…)

Digital modulation : $A_i, \theta_i, f_i(t)$ are discrete-valued (PSK,FSK,QAM,…)

Modulation Waveforms in Cellar Systems

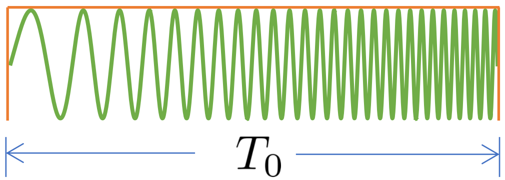
Cellular Evolution	1G (1980's)	2G (1990's)	3G (2000's)	4G (2010's)	5G (2020's)
Data Rate	2.4 kbps	64 kbps	100 kbps - 56 Mbps	Up to 1 Gbps	> 1 Gbps
f_c	800-900 MHz	850-1900MHz	1.6-2.5GHz	2-8 GHz	Sub- 6GHz, mmWave
Modulation	FDMA	TDMA	CDMA	OFDM	OFDM
Pulse/Filter	N/A	Gaussian	RRC (chip) pulses modulated by spreading code	Complex-Sinusoids/ Subcarriers/Tones truncated by rectangular pulse	Complex-Sinusoids/ Subcarriers/Tones truncated by rectangular pulse
$e^{j2\pi f_i(t)t}p_i(t)$	N/A	$g_i(t)$  $f_i(t)=0$	$g_i(t)$  $f_i(t)=0$	$g_i(t)$  $f_i(t)=n\Delta f$	$g_i(t)$  $f_i(t)=n\Delta f$

- Modulations are designed to achieve **high transmission efficiency** and to deal with **fading and interference**.
- OFDM is fundamentally designed for linear time-invariant (LTI) channels
- OFDM has difficulty coping with doubly selective linear time-varying (LTV) channels (delay and Doppler effects)

Radar Waveforms

Constant envelope **continuous** wave：

FMCW (Frequency Modulated Continuous Wave)



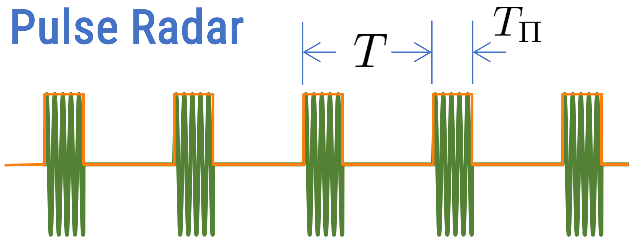
$$\sqrt{2}A\cos\left(2\pi(f_c + f_0 + kt)t + \theta + \theta_c\right)p(t)$$

Rectangular pulse (duration T_0, bandwidth $\propto 1/T_0$) → $p(t)$

Chirp rate $(k \gg 1/T_0^2 \Rightarrow kT_0 \gg 1/T_0)$ → k

Constant envelope **non-continuous** wave：

Pulse Radar



$$\sqrt{2}A\cos\left(2\pi f_c t + \theta + \theta_c\right)p(t) \qquad p(t) = \sum_{n=0}^{N-1} \Pi(t - nT)$$

Pulse train → $p(t)$

Rectangular pulse (duration T_Π, bandwidth $\propto 1/T_\Pi$) → Π

Pulse interval $(T \gg T_\Pi)$ → T

- Aiming for high delay-Doppler resolution, i.e., high time-frequency (TF) resolution
  - FMCW：Basically, FM signal. Because of $kT_0 \gg 1/T_0$, transmission efficiency is not high
  - Pulse Radar：No signal transmission between two adjacent pulses ($T \gg T_\Pi$), transmission efficiency is low.

ISAC Waveform Consideration

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Modulation	Radar
Mutually orthogonal pulses that match to channel characteristics	Pulse with high TF resolution

- ✓ For ISAC, we prefer orthogonal pulses whose orthogonality is defined with respect to high TF resolution.

- ➢ The highest TF resolution is the delay-Doppler (DD) resolution.
- ➢ Any channel characteristics associated with the DD resolution?

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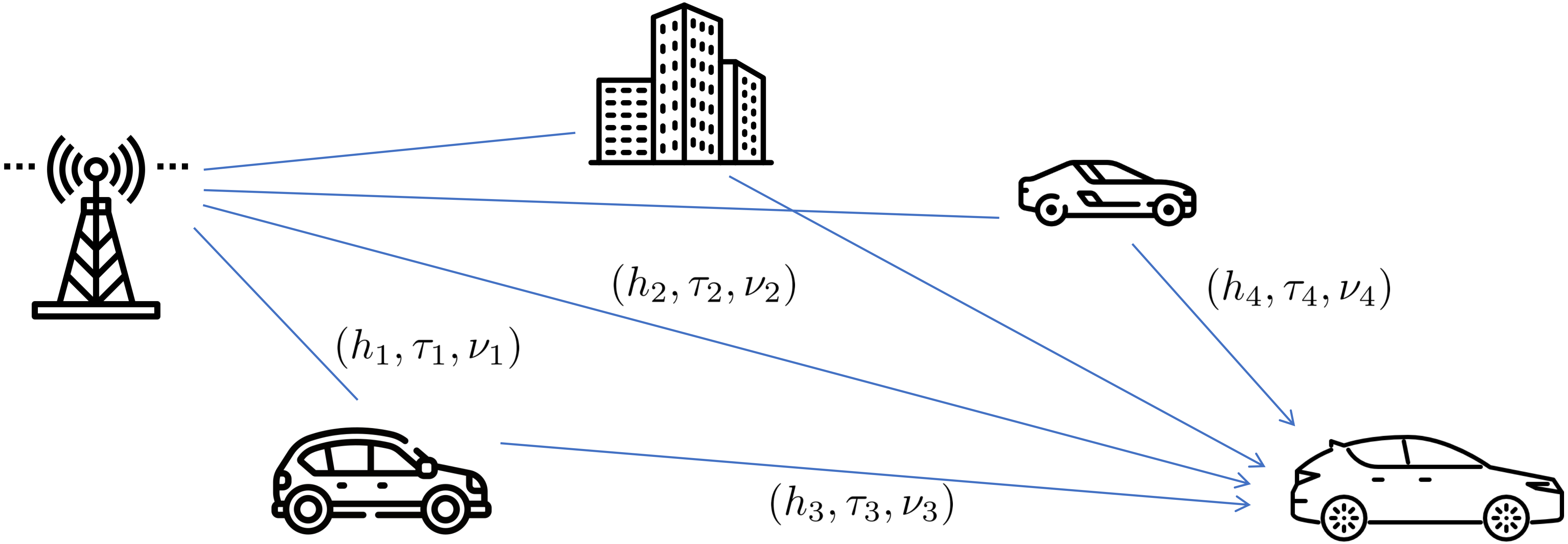
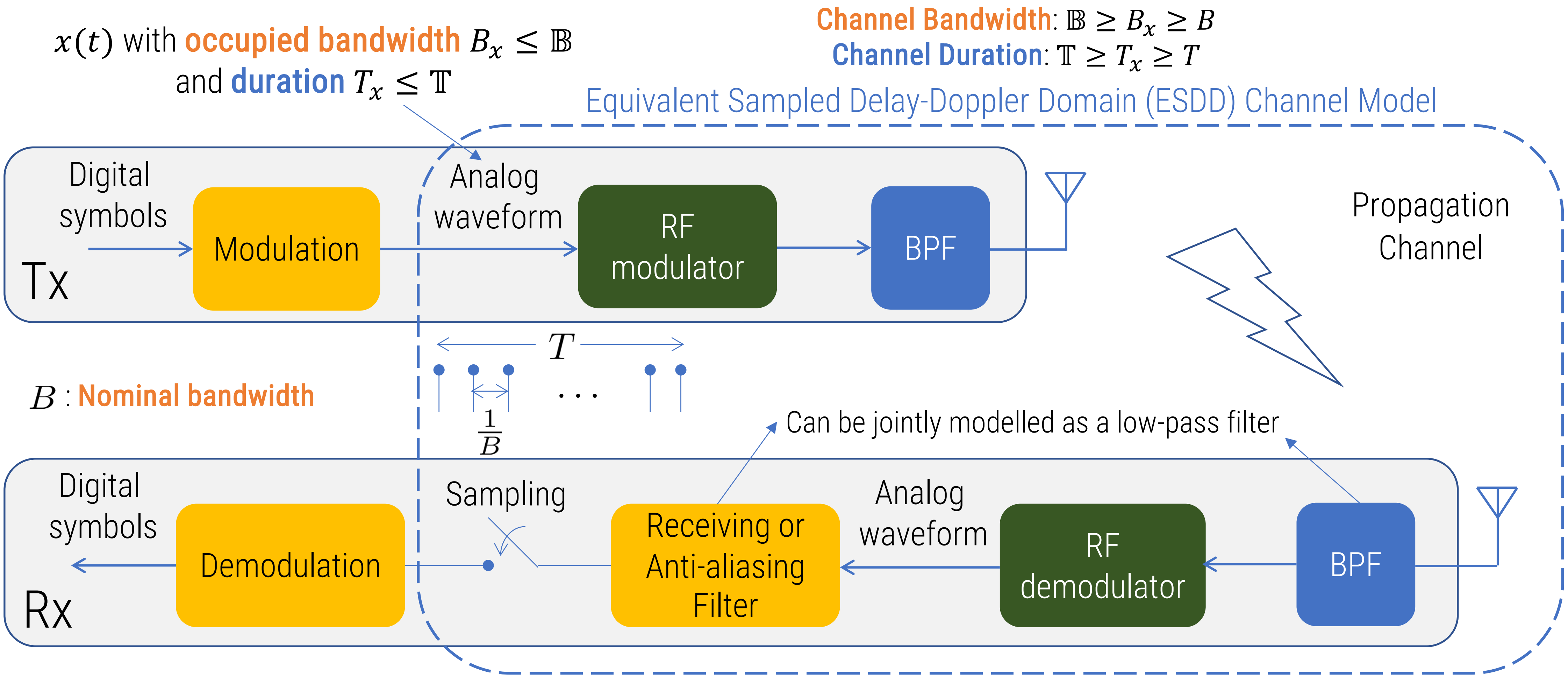
Mobile Channel Models



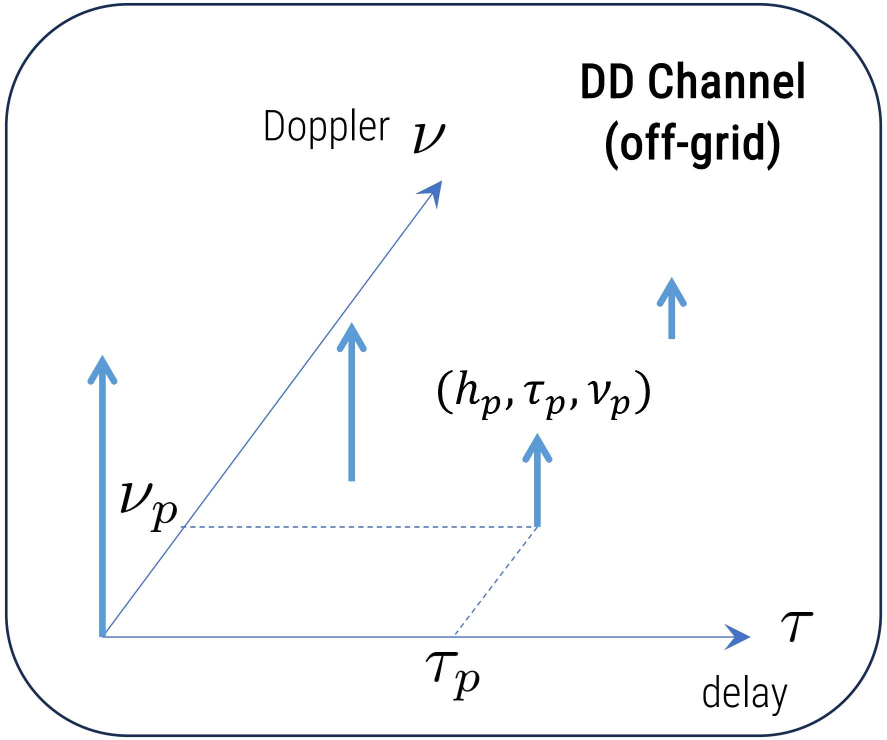
- Doubly-selective channel with both time and frequency dispersions
- Statistical models: WSSUS, Rayleigh, Rician, Nakagami-m
- Deterministic model: delay-Doppler spread function, namely spreading function $\mathcal{S}(\tau, \nu), h(\tau, \nu)$

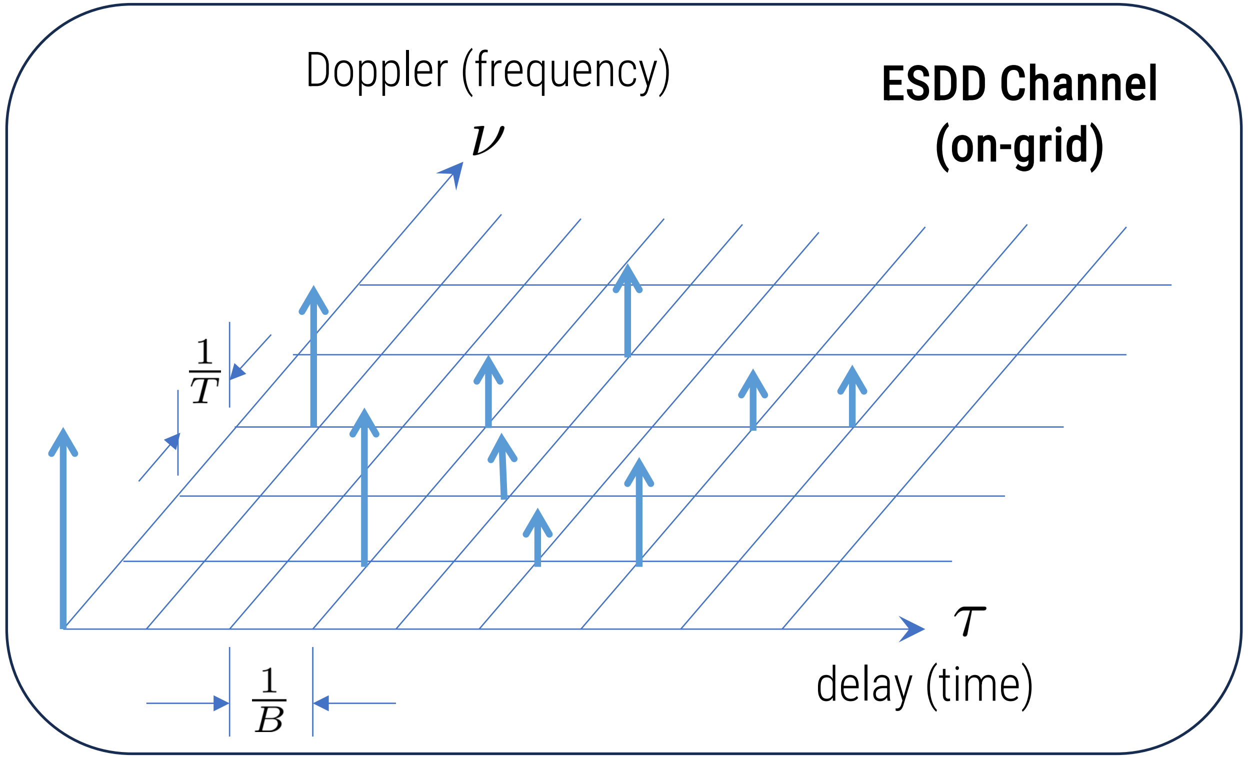
DD Domain in Practical Systems



- P. Bello, "Characterization of randomly time-variant linear channels," IEEE Trans. Commun. Syst., vol. 11, no. 4, pp. 360–393, 1963.

On-Grid DD Domain in Practice





- In practice, we observe an on-grid DD domain, due to the limited bandwidth and duration of signal.
- Physical unit of delay and Doppler are time and frequency, respectively.
- On-grid DD domain is exactly an on-grid TF domain : **Frequency grid -> Multi-Carrier Waveform**
- DD domain waveform in practice is a **DD domain multi-carrier (DDMC) waveform**

Common Channel Characteristics

- For communications, the best choice of pulses may be **the eigenfunctions of the channel**.
- The underspread LTV/DD channels at best have a structured set of approximate eigenfunctions, which are even **channel-dependent (not common)**.
  - W. Kozek and A. Molisch, "On the eigenstructure of underspread WSSUS channels," in Proc. IEEE SPAWC, 1997, pp. 325−328.
- Because DD resolution is **determined by the signal**, ESDD Channels have a **common DD resolution** $(\frac{1}{B}, \frac{1}{T})$.

	LTI Channels	LTV/DD Channels
Common Characteristics	Bandwidth $\mathbb{B} \geq B$ Duration $\mathbb{T} \geq T$ Delay resolution: $\frac{1}{B}$ Eigenfunction $e^{j2\pi ft}, -\infty \leq t \leq +\infty$	Bandwidth $\mathbb{B} \geq B$ Duration $\mathbb{T} \geq T$ **DD resolution**: $(\frac{1}{B}, \frac{1}{T})$

- DD domain waveform : **Design an MC waveform according to the DD resolution** $(\frac{1}{B}, \frac{1}{T})$

Waveform Comparison

Waveform	Single Carrier	CDMA	OFDM	FMCW	Pulse Radar
Channel characteristics / pulse shape used	Bandwidth / RRC pulse	RRC pulse train modulated by spreading code	Eigenfunction of LTI channel /truncated complex sinusoids	Chirp	Rectangular pulse train
Transmission Efficiency	○	○	○	×	×
Equalization (LTI Channel)	△	○	◎	○	○
Equalization (LTV Channel)	△	△	△	○	○
Sensing/Radar (LTV Channel)	△	△	△	◎	◎

- **DDMC** may enable both channel-matched communication and high TF resolution.

Multi-Carrier (MC) Modulation

Such pulses/functions are known as Weyl Heisenberg (WH) /Gabor set

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} X[m,n] g(t - m\Delta T) e^{j2\pi n\Delta F(t-m\Delta T)}$$

- $X[m,n]$: Digital symbol / number drawn from a constellation
- $g(t - m\Delta T)$: Prototype pulse usually orthogonal w.r.t. $\Delta T, \Delta F$
- $e^{j2\pi n\Delta F(t-m\Delta T)}$: Subcarrier/Tone, a.k.a. eigenfunction of LTI channel

- $x(t)$ is well known as **OFDM** for **rectangular** $g(t)$ and **pulse-shaped (PS) OFDM** for **non-rectangular** $g(t)$.
- At the Rx, **matched filtering** (or **correlators**) → extract digital symbols → equalization.
- Prefer **(bi)orthogonal** pulses that remain orthogonal even after channel distortion

- G. Matz, H. Bolcskei, and F. Hlawatsch, “Time-frequency foundations of communications: Concepts and tools,” IEEE Signal Process. Mag., vol. 30, no. 6, pp. 87–96, 2013.
- B. Le Floch, M. Alard and C. Berrou, "Coded orthogonal frequency division multiplex," Proc. IEEE, vol. 83, no. 6, pp. 982-996, 1995.

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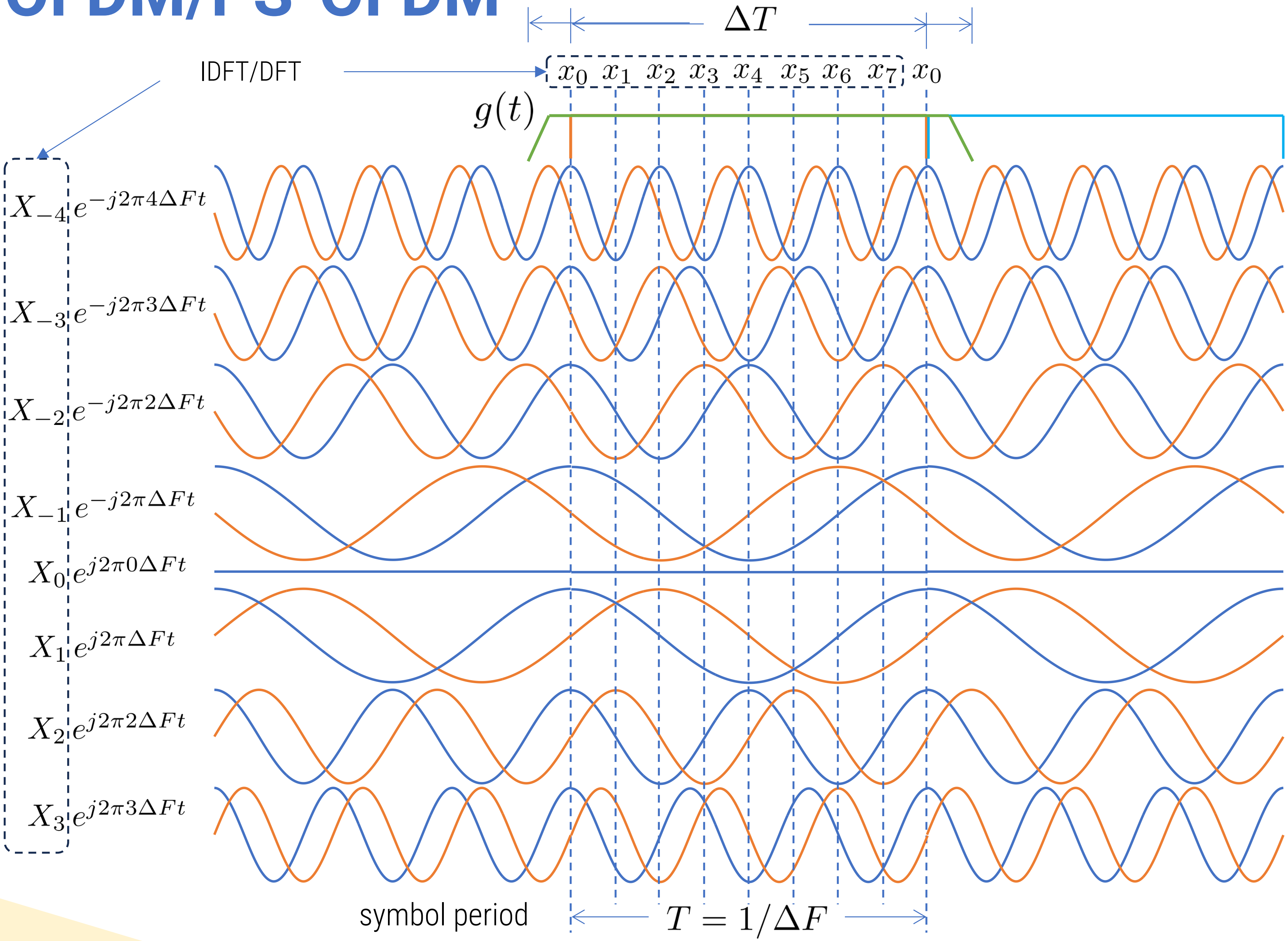
OFDM/PS-OFDM



1. Given ΔF, we have symbol period $T = 1/\Delta F$
2. For ΔT, find $g(t)$ (OFDM/PS-OFDM)
3. Symbol period T, symbol interval ΔT , and symbol duration T_g are **three different parameters**!
4. Truncate the subcarriers using $g(t)$
5. IDFT-based implementation

It is all about Ambiguity Function!

- Ambiguity function (AF): Cross-correlation of a pulse $g(t)$ and its TF-shifted version given by

$$A_{g,g}(\tau,\nu) = \langle g(t), g_{\tau,\nu}(t)\rangle = \int_{-\infty}^{+\infty} g^*(t)g(t-\tau)e^{j2\pi\nu(t-\tau)}dt$$

- In MC modulation design, we aim for **orthogonality or (bi)orthogonality with respect to** the TF resolution $(\Delta T, \Delta F)$

$$A_{g,g}(m\Delta T, n\Delta F) = \langle g(t), g_{m,n}(t)\rangle = \int_{-\infty}^{+\infty} g^*(t)g(t-m\Delta T)e^{j2\pi n\Delta F(t-m\Delta T)}dt = \delta(m)\delta(n)$$

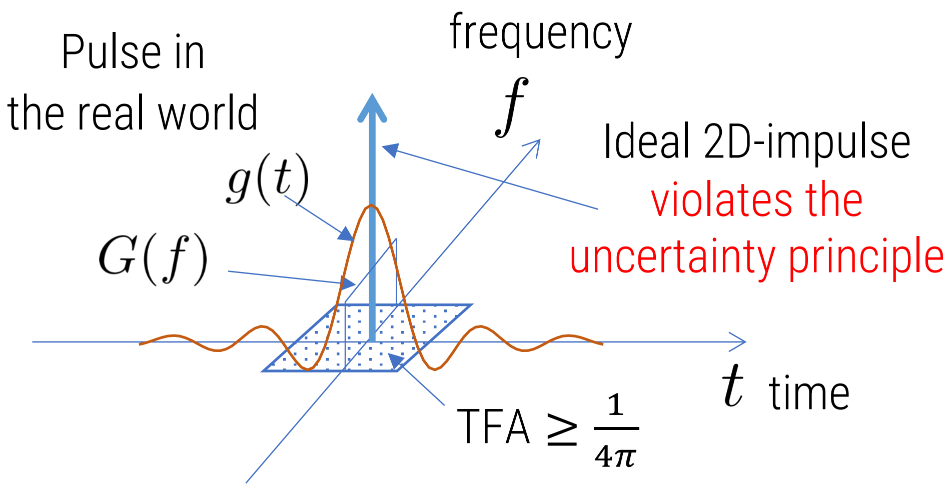
- In a general off-grid DD channel, a propagation path (h_p, τ_p, ν_p) TF-translates the pulse to $h_p g(t-\tau_p)e^{j2\pi\nu_p(t-\tau_p)}$
- Then, with the receive pulse $r(t)$, the receiver performance depends on the AF

$$A_{r,g}(\tau_p,\nu_p) = \langle r(t), g_{\tau_p,\nu_p}(t)\rangle = \int_{-\infty}^{+\infty} r^*(t)g(t-\tau_p)e^{j2\pi\nu_p(t-\tau_p)}dt$$

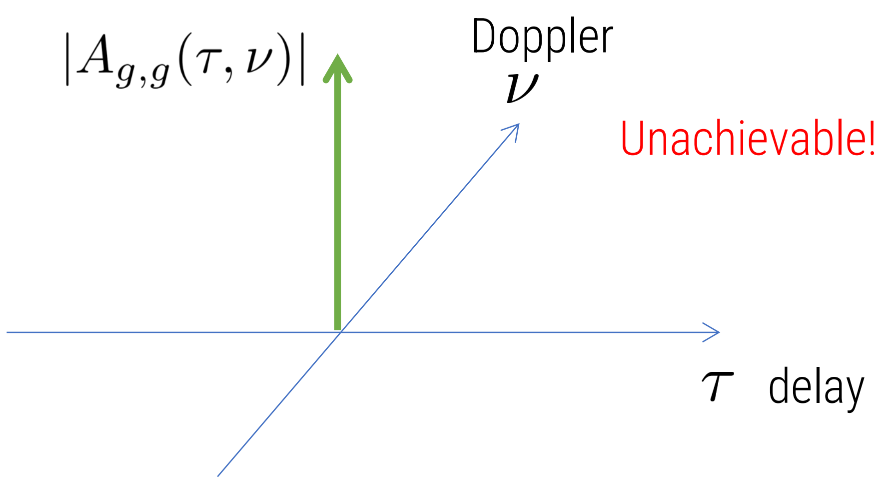
- For radar or sensing application, the best pulse is **the ideal TF/DD domain 2D-impulse that yields an impulsive AF**

$$A_{g,g}(\tau,\nu) = \delta(\tau)\delta(\nu) \quad \text{or} \quad A_{r,g}(\tau,\nu) = \delta(\tau)\delta(\nu)$$

Uncertainty Principle



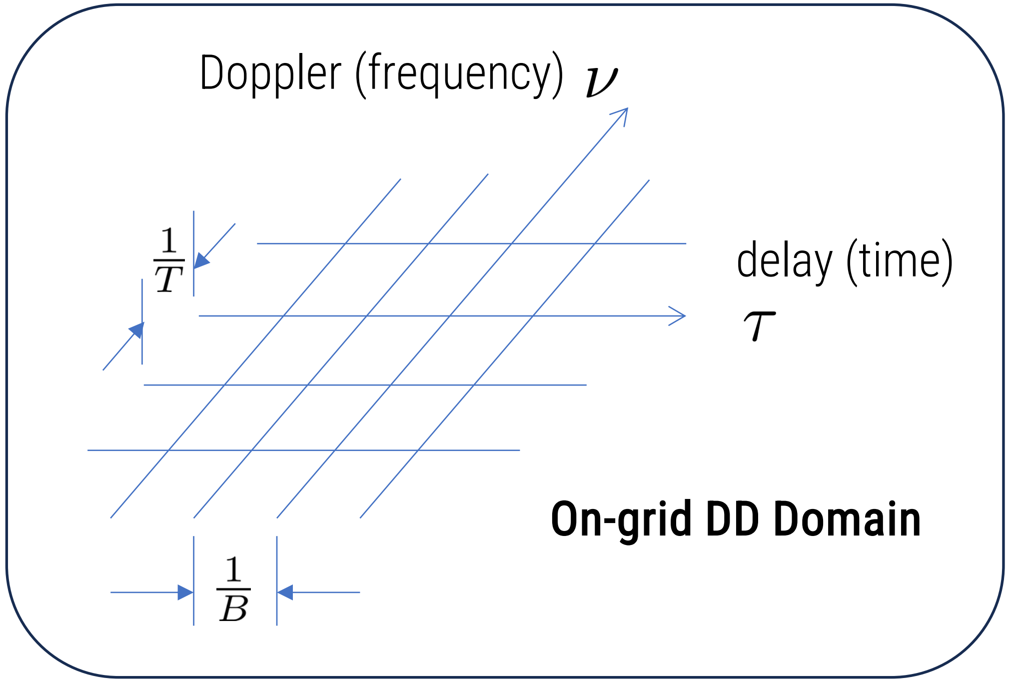
Inter-dependent TF domain for pulse design

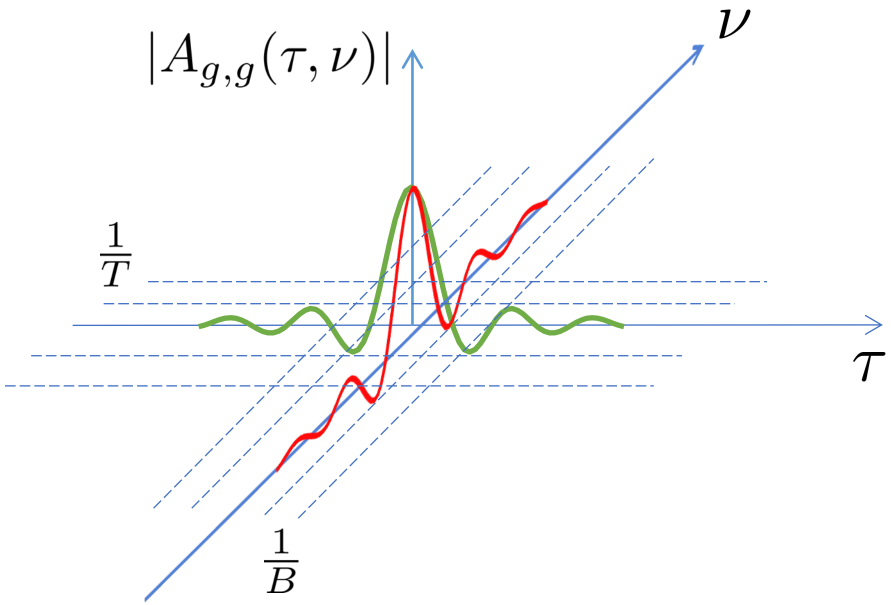


Impulsive AF of ideal 2D-Impulse

- A pulse in the real world is always a 1D function, represented in either the time or the frequency domain.
- Let $\alpha T_g, \alpha B_g$ be the standard deviations of $g(t)$ and $G(f)$, respectively. $\alpha T_g \times \alpha B_g$ is called the TF area (TFA) .
- According to the **uncertainty principle**, the TFA obeys a lower bound $\alpha T_g \alpha B_g \geq \frac{1}{4\pi} \approx 0.0796$.
- The ideal 2D-impulse **does not exist,** and the impulsive AF is **unachievable**.

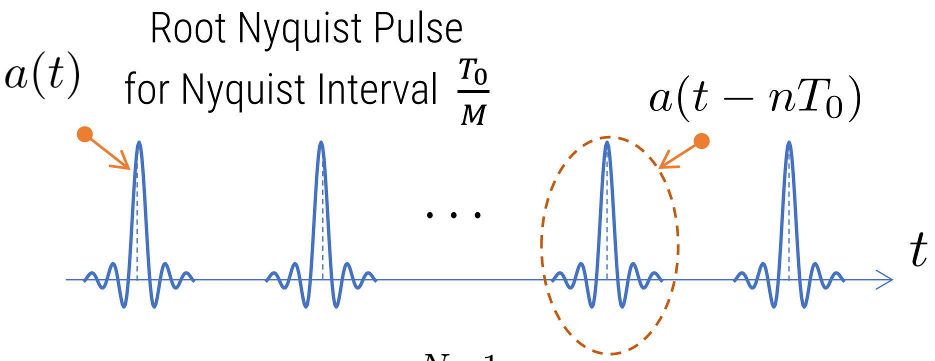
On-Grid Ambiguity Function



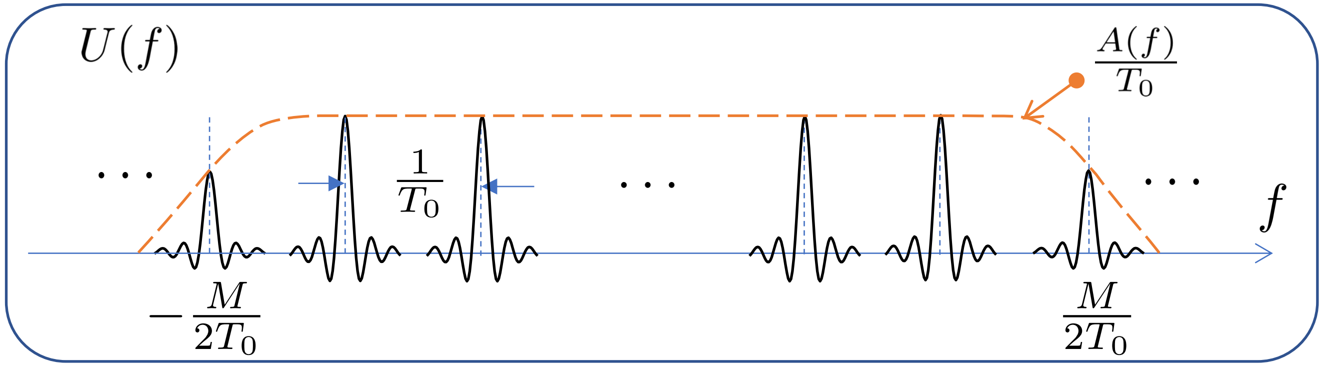


- For sensing applications, we usually aim to suppress the AF-based integrated side-lobe level (ISLL).
- Given the on-grid DD domain in practice, should we instead consider the on-grid AF ?
- Is there any pulse that can achieve the **minimum ISLL of the on-grid AF**? **Yes!**

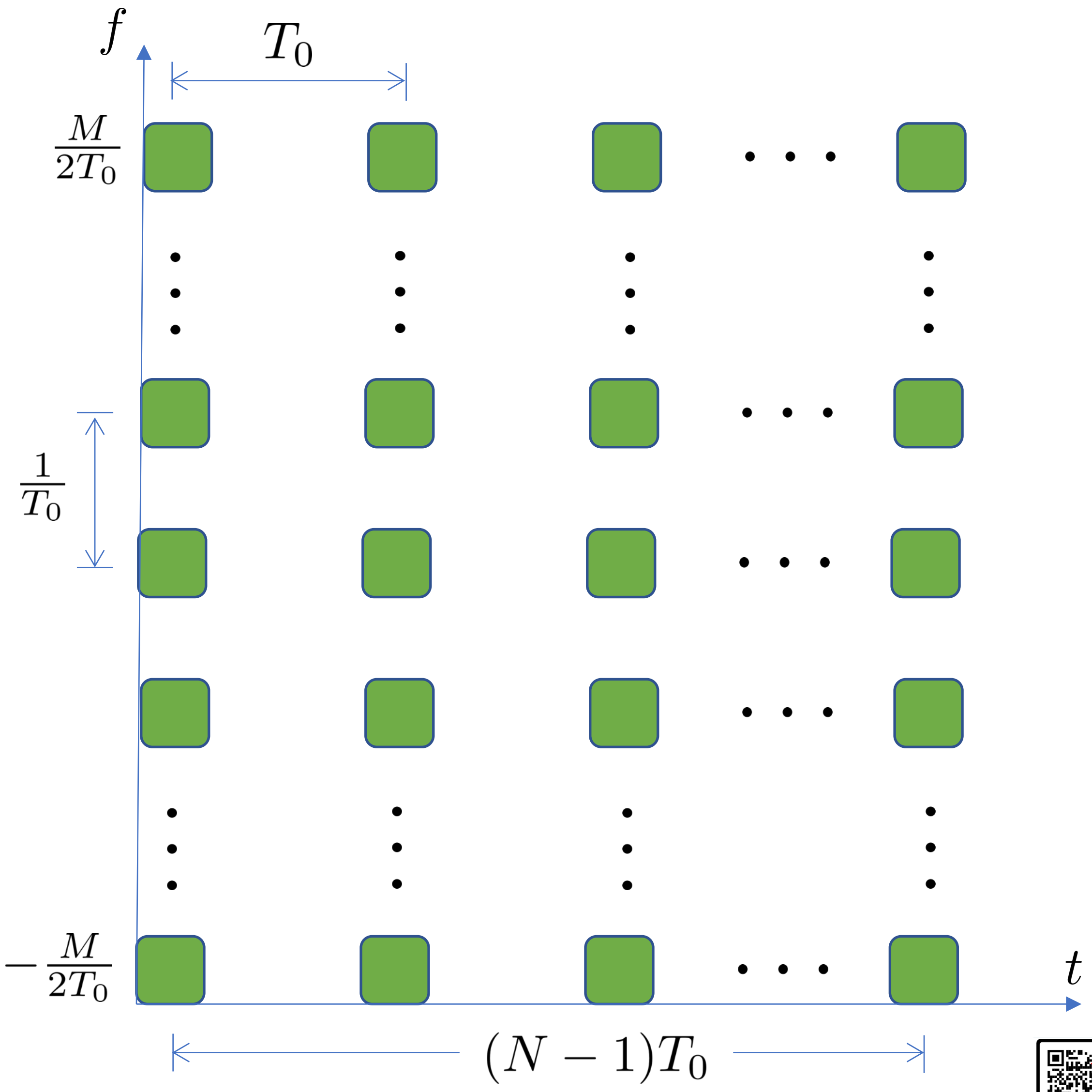
DD Domain Orthogonal Pulse (DDOP)



$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0)$$

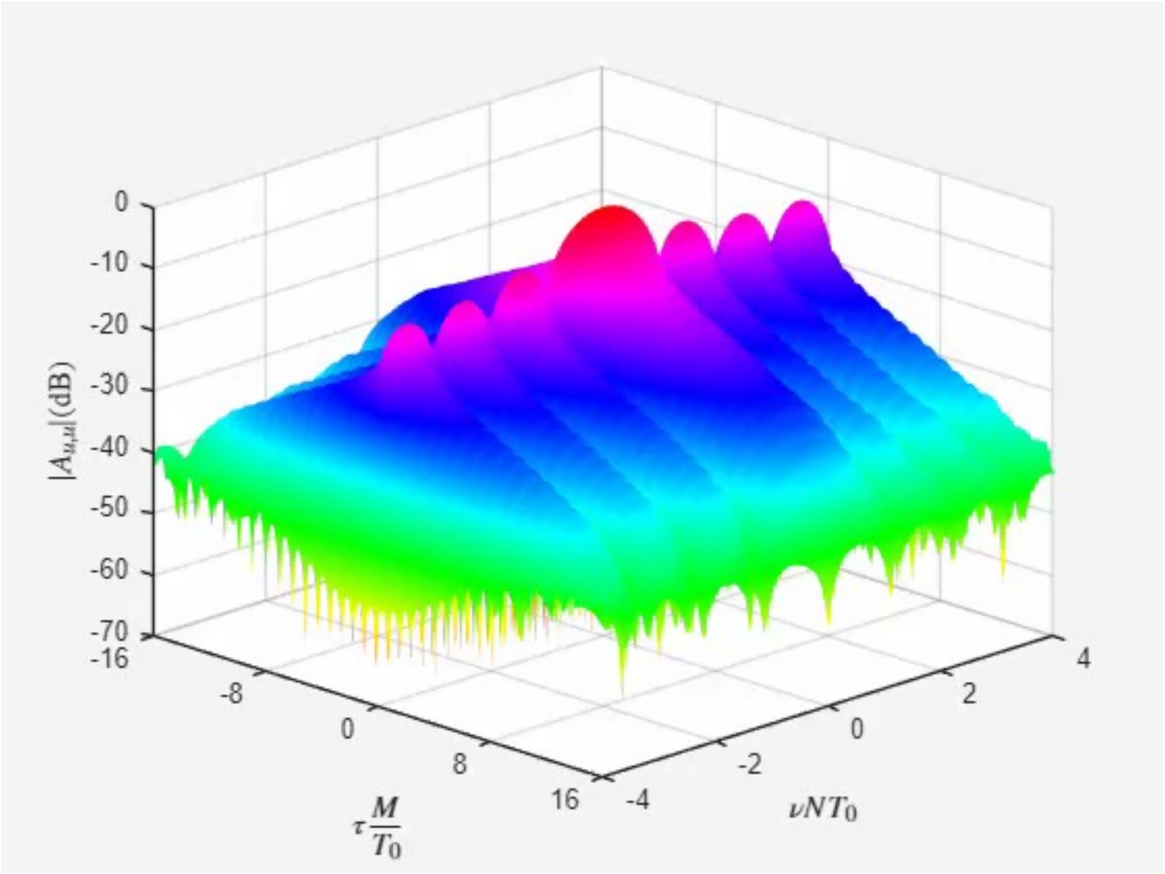


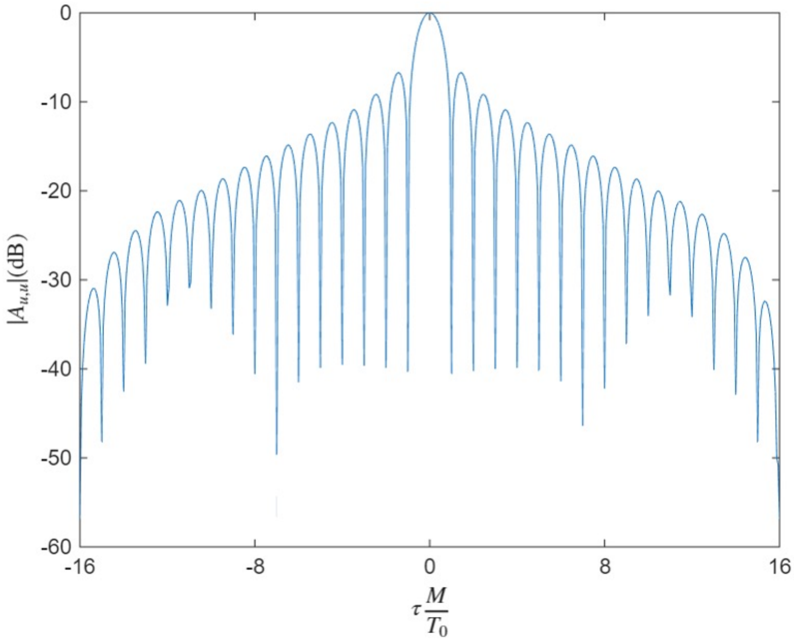
$$U(f) = \frac{e^{-j2\pi f\tilde{T}}}{T_0} A(f) \sum_{m=-\infty}^{\infty} e^{j2\pi \frac{m(N-1)}{2}} \operatorname{Sinc}(fNT_0 - mN)$$

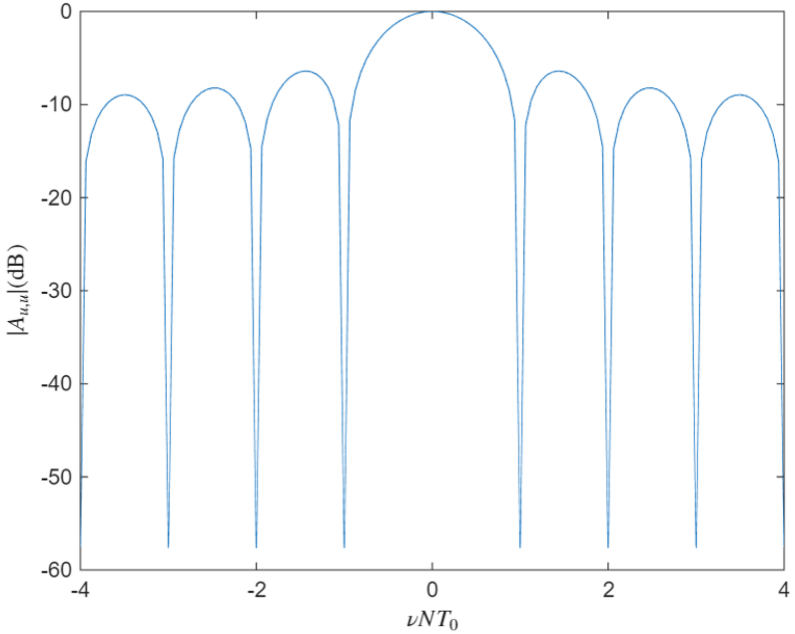


DDOP's Ambiguity Function

$$A_{u,u}\left(m\frac{T_0}{M}, n\frac{1}{NT_0}\right) = \delta(m)\delta(n), \forall |m| \leq M-1, |n| \leq N-1$$

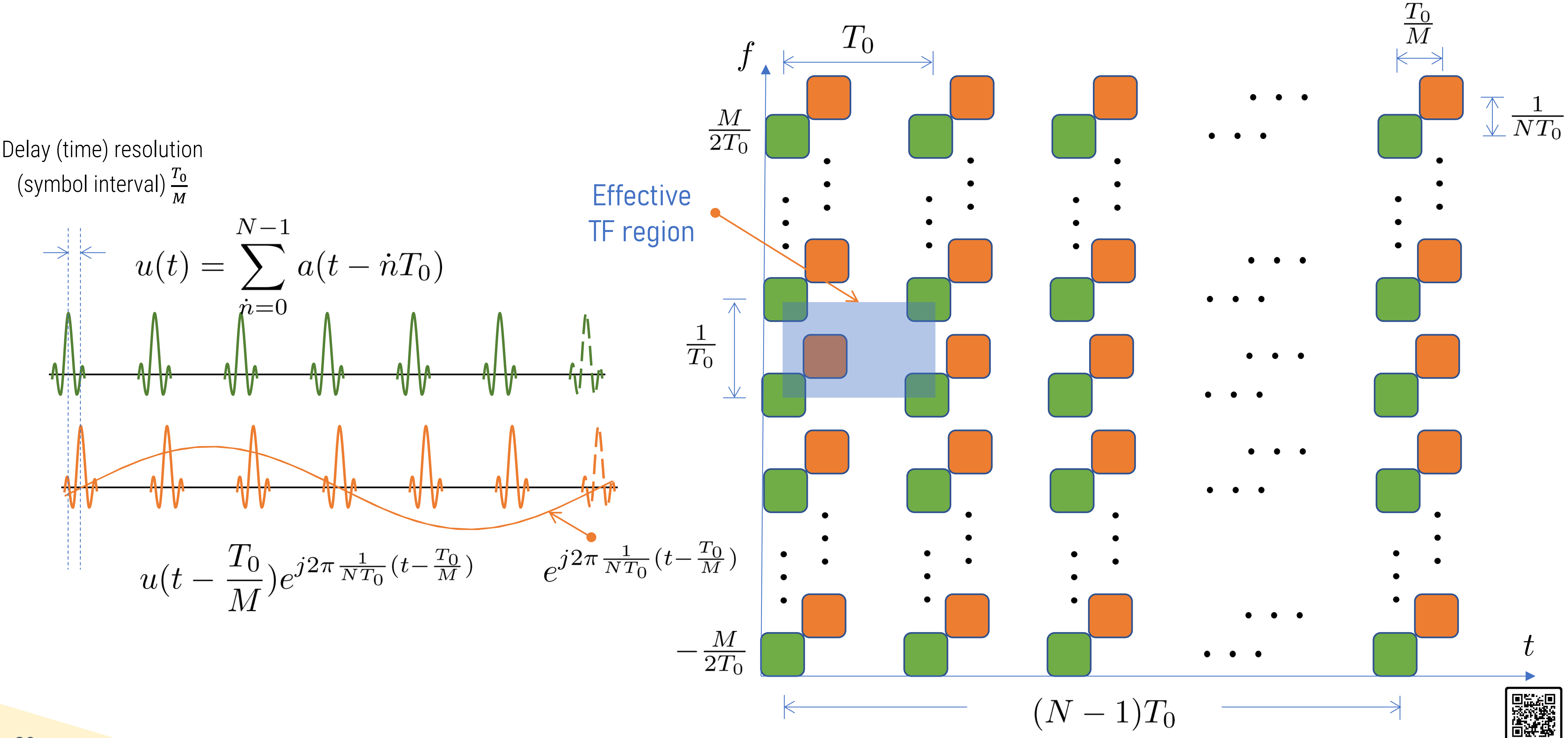




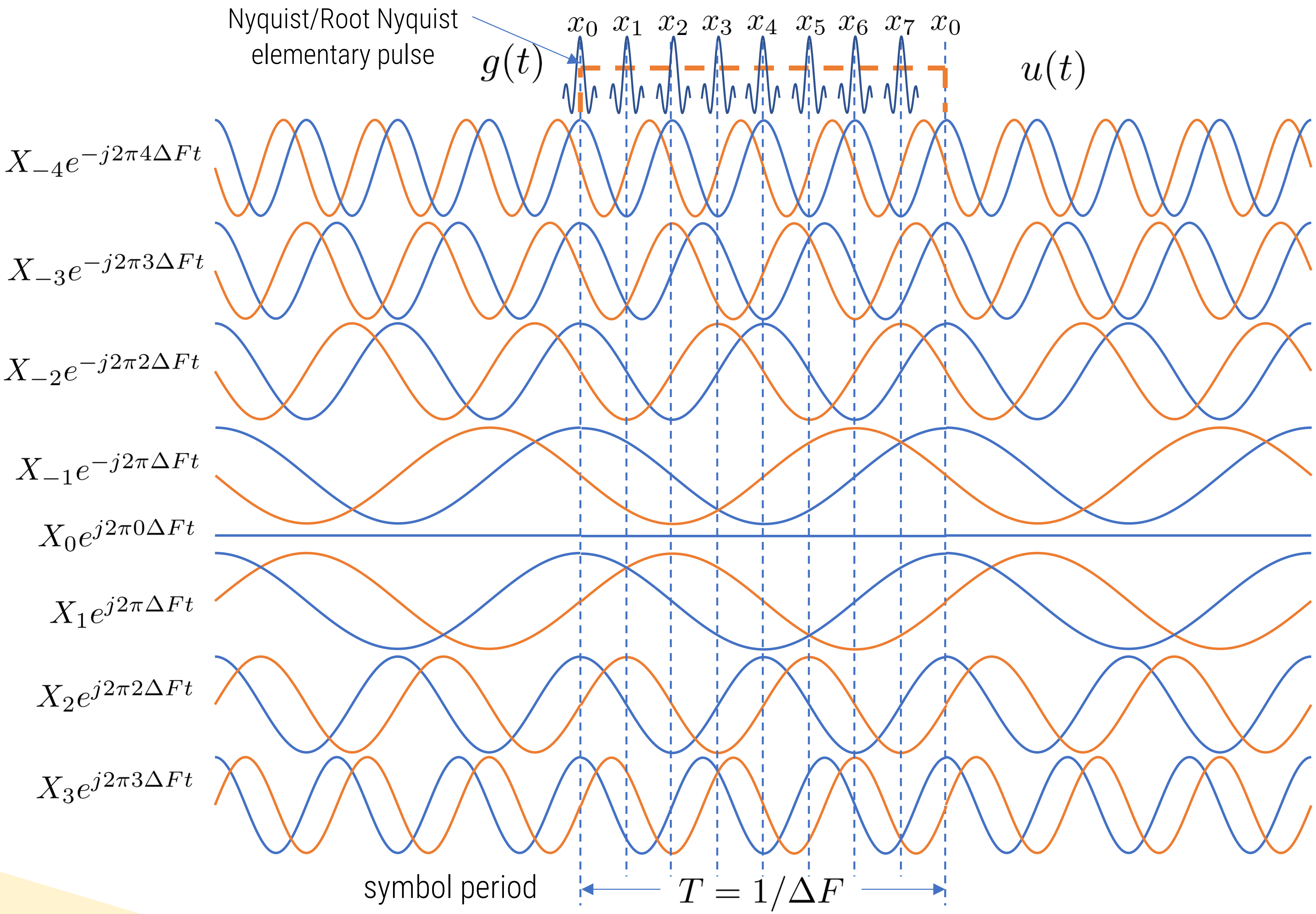


$$M = 32, N = 8, \rho = 0.1$$

TF-Translated DDOP



Orthogonal Delay-Doppler Division Multiplexing (ODDM)



- DD domain/plane orthogonal pulse **(DDOP)**
- A pulse-train can achieve the orthogonality among subcarriers
- Orthogonality among symbols can be achieved by employing Nyquist/Root Nyquist elementary pulse.
- WH subset based waveform design

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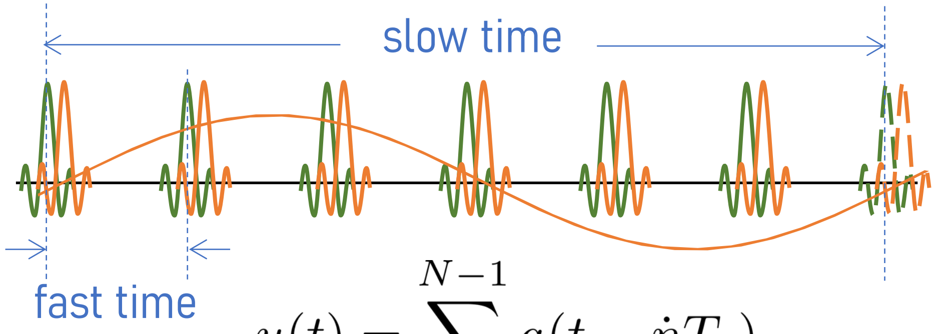
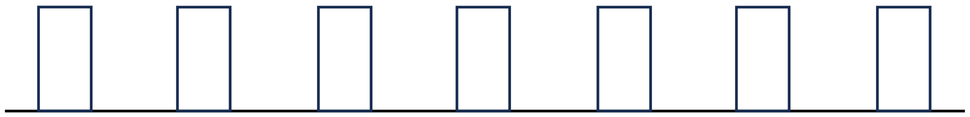
Underlying Logic of DDOP

	DD domain 2D-impulse	DDOP
Time duration	Short	Long
Frequency bandwidth	Narrow	Wide
Uncertainty Principle	Violates, doesn't exist	Doesn't violate, exist

Pseudo-2D-impulse in on-grid DD domain !



$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0)$$



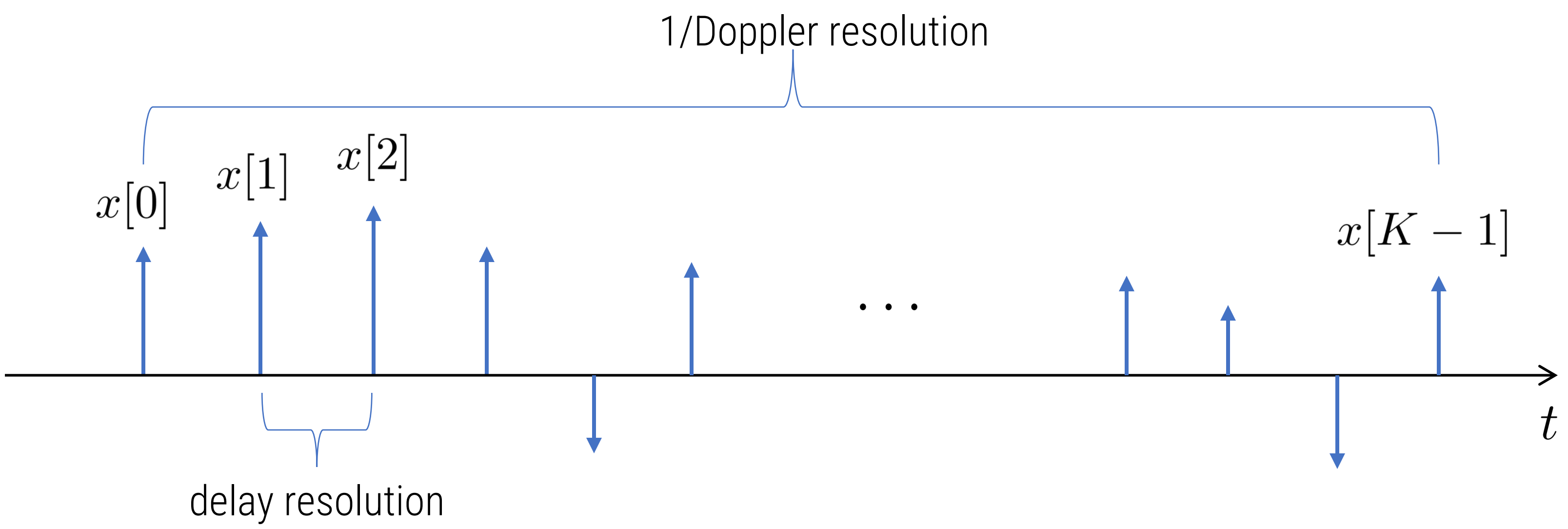
$$g(t) = \sum_{n=0}^{N-1} \Pi(t - nT)$$

- DDOP is a type of pulse radar.
- By **staggering symbols**, transmission efficiency is high.
- To suppress ISI, root Nyquist pulse is adopted in the pulse train.

} Combines key characteristics of radar and modulation waveforms

From the Perspective of Sequence-based Waveform Design

- A popular approach to waveform design is based on **sequence optimization**.



- Discrete-time sequence ⇒ **DD resolution fixed**
- Sequence-based AF = **on-grid AF**
- Sequence optimization to suppress ISLL ⟶ based on on-grid AF
- Minimum on-grid-AF-based ISLL can be **achieved by the DDOP** (**within corresponding DD range**)

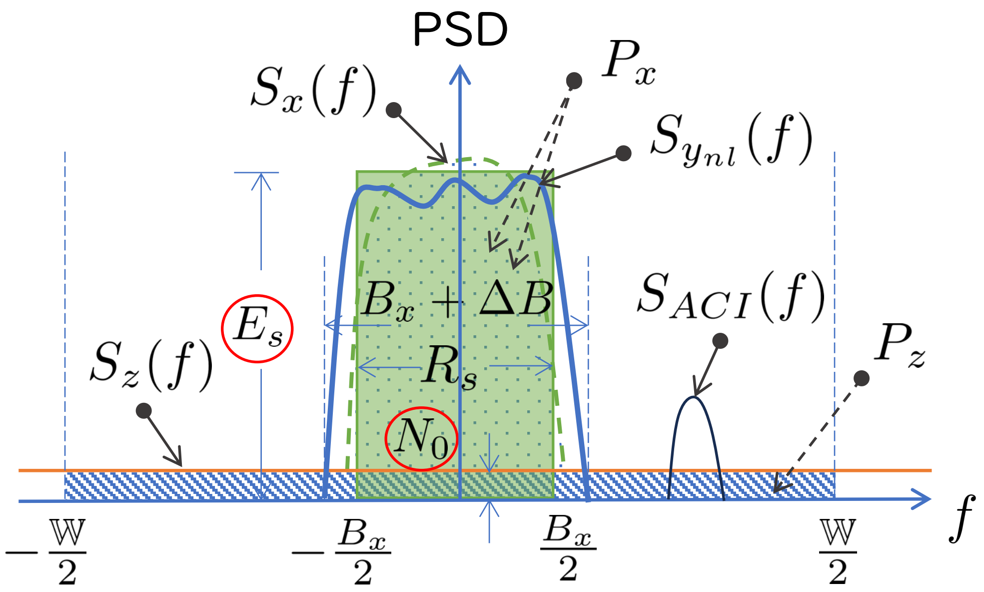
Waveform-Level Simulations

- Waveform-level simulations are **crucial for comparing waveforms with different bandwidths**.
- The received signal is a superposition of the noiseless channel output and AWGN.

$$y(t) = y_{nl}(t) + z(t)$$

Ideal AWGN (**infinite bandwidth and therefore infinite power**)

- Discrete-time signal for simulation : $y[k] = y_{nl}[k] + z[k]$
  - Sampling interval : T_s
  - **Anti-aliasing filter bandwidth** : $\mathbb{W} = \frac{1}{T_s}$
- Waveform-level simulation : **oversampling-based**
  - Simulation bandwidth : $\mathbb{W} = \frac{1}{T_s} \gg B_x + \Delta B$
- Noise samples $z[k]$ have **high power** : $P_z = N_0 \mathbb{W}$
- Since the SNR depends on $\mathbb{W}$, we prefer $\frac{E_b}{N_0} = \frac{E_s / \log_2 M_X}{N_0}$.



- The simulation results should be independent of $\mathbb{W}$, as long as $\mathbb{W} = \frac{1}{T_s} \gg B_x + \Delta B$.

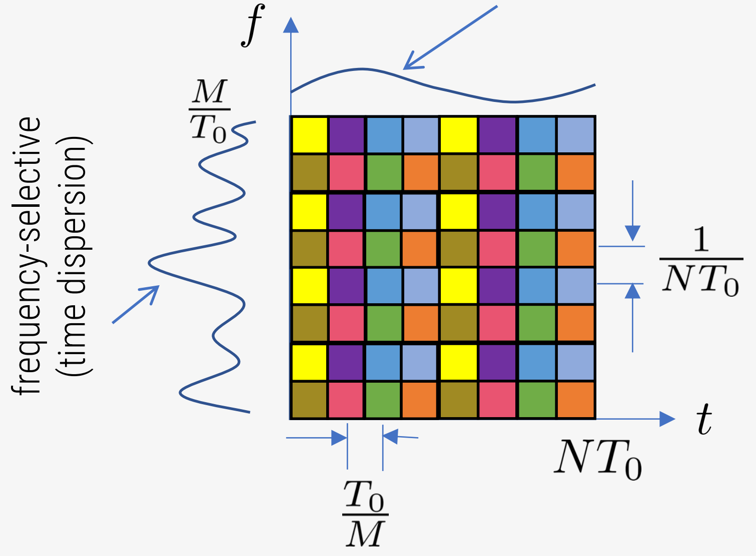
Conclusion

- ➢ DD domain orthogonal pulse (DDOP)
  - ➢ **Pseudo 2D impulse in on-grid DD domain**
  - ➢ Pulse train with long duration and wide bandwidth
  - ➢ "Circumvent" the limits of the uncertainty principle

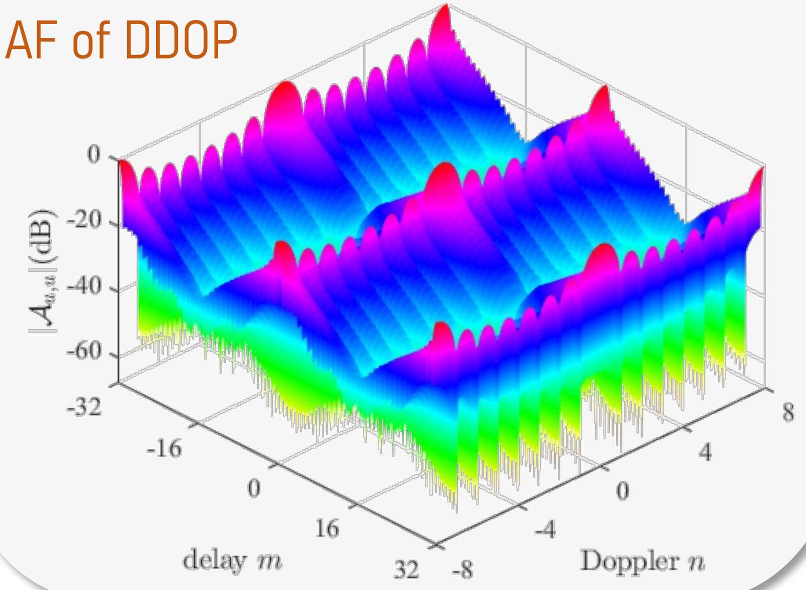
- ➢ DDOP-based ODDM waveform
  - ✓ A promising waveform candidate for ISAC
  - ✓ Embracing DD channel property

- ➢ Many open issues. More details at: https://oddm.io

TFEL of ODDM



AF of DDOP



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- For more information, please Google/Bing “ODDM” or visit **https://oddm.io**

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Acknowledgment

➢ Thanks to my collaborators

Prof. Jinhong Yuan, the University of New South Wales, Australia

Prof. Wei Yu, the University of Toronto, Canada

Prof. Jingxian Wu, University of Arkansas, USA

Prof. Lajos Hanzo, University of Southampton, UK

➢ Thanks to my funding bodies, industry partners, and great students

■ Questions or Comments : Please send to hai.lin@ieee.org

Thank you for your attention!

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