On Delay-Doppler Plane Orthogonal Pulse

Hai Lin[*] and Jinhong Yuan[**]

* Osaka Metropolitan University, Sakai, Osaka, 599-8531, Japan

** The University of New South Wales, Sydney, NSW, 2052, Australia

IEEE GLOBECOM 2022, 7 December 2022, Rio de Janeiro, Brazil

Multicarrier (MC) Modulation

- MC Modulation

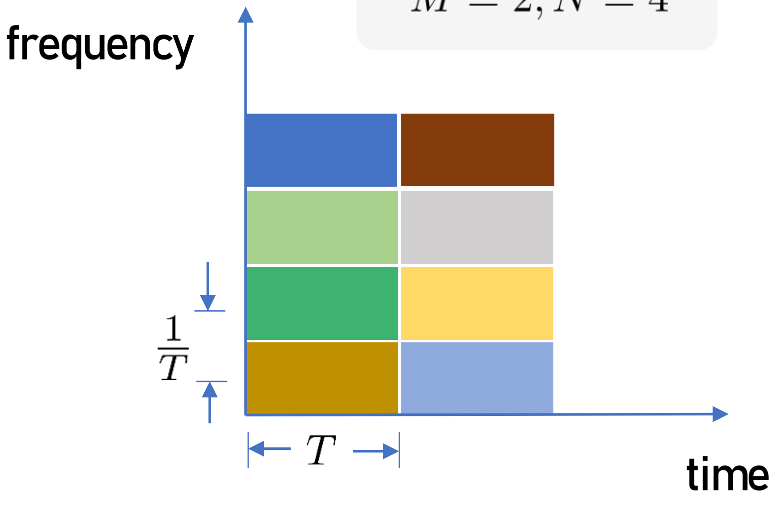
Transmit Pulses (Filters)

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X_{m,n} g(t - m\mathcal{T}) e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$

Information-bearing digital symbols

TF Lattice

$M = 2, N = 4$

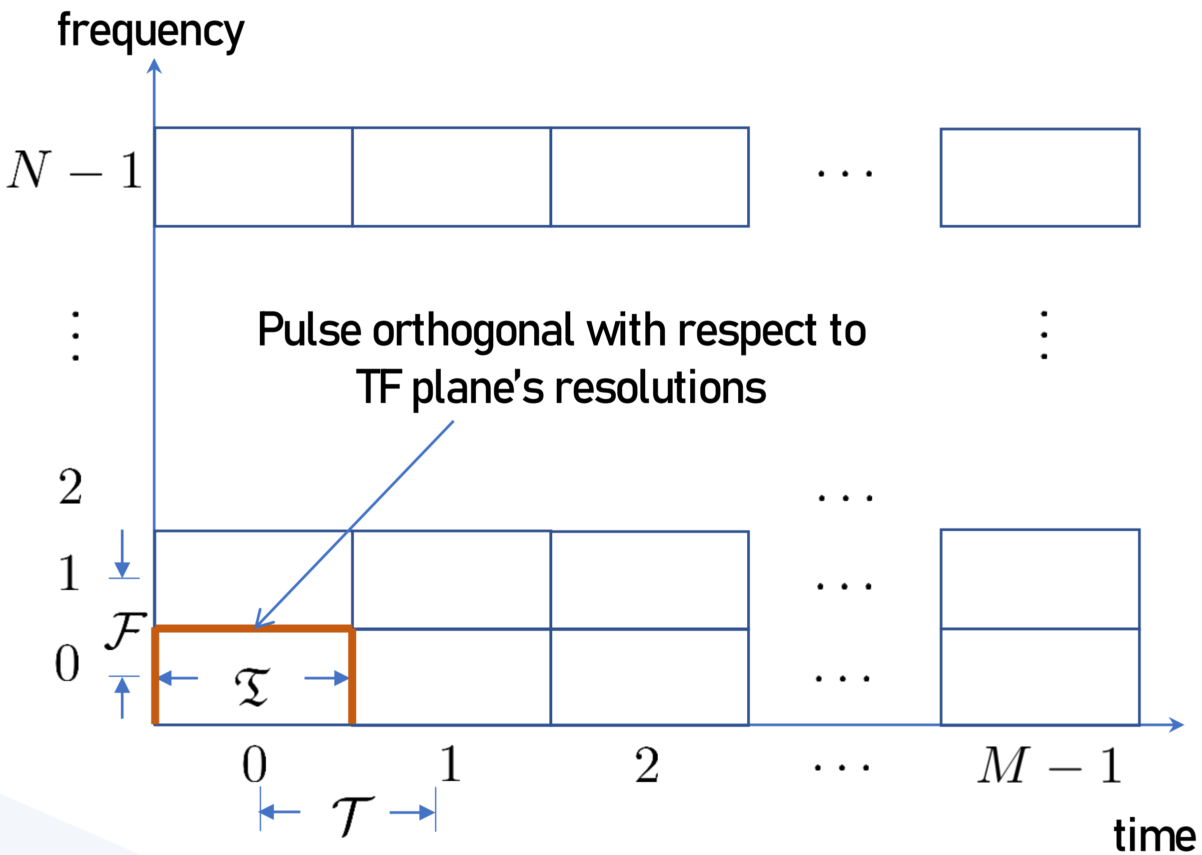


TF plane

- Pulse is a fundamental element of any modulation
  - defines the shape of signal in both time and frequency domains
  - determines how energy spread over the time and frequency
  - affects signal characteristics significantly (efficiency, localization, orthogonality, etc.)
- This paper
  - Design orthogonal pulses w.r.t delay-Doppler resolutions

Pulse Design Principles

- One pulse for one symbol
- MN symbols ⇒ MN pulses orthogonal with each other



TF localization structure

time resolution (symbol interval)

frequency resolution (subcarrier spacing)

$$g_{m,n}(t) = g(t - m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$
$$0 \le m \le M-1, 0 \le n \le N-1$$

Joint TF resolution (JTFR) : $\mathcal{R} = \mathcal{T}\mathcal{F}$

Symbol period : $\mathfrak{T} = \frac{1}{\mathcal{F}}$

- <u>Core of MC modulation</u>: $g(t)$ given $(\mathcal{T}, \mathcal{F})$

Pulse Design Principles

- Core of MC modulation: **prototype pulse/filter** $g(t)$ given $(\mathcal{T}, \mathcal{F})$
- Traditionally, $g_{m,n}(t)$'s are treated as **Weyl-Heisenberg (WH)** or **Gabor** function set
- Design an MC modulation

  ⇒ Find (bi)orthogonal WH/Gabor sets ⇒ Find $\boldsymbol{g(t)}$ given $(\boldsymbol{\mathcal{T}}, \boldsymbol{\mathcal{F}})$

- Most of orthogonal MC modulation schemes are designed with $\boldsymbol{\mathcal{R} = \mathcal{T}\mathcal{F} \geq 1}$
- OFDM: Rectangular pulse for $\mathcal{R} = \mathcal{T}\mathcal{F} = 1$, namely time resolution = symbol period

- G. Matz, H. Bolcskei, and F. Hlawatsch, "Time-frequency foundations of communications: Concepts and tools," IEEE Signal Process. Mag., 2013.
- A. Sahin, I. Guvenc, and H. Arslan, "A survey on multicarrier communications: Prototype filters, lattice structures, and implementation aspects," IEEE Commun. Surveys Tuts., 2014.

(Bi)Orthogonal WH/Gabor Sets

- Fundamental tool of time-frequency analysis（TFA）for signals/functions
- Gabor (Weyl-Heisenberg, Short-time/Windowed Fourier) expansion
- For signals lie in space $L^2(\mathbb{R})$

$$x(t) = \sum_{m\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} c_{m,n} g_{m,n}(t), \quad g_{m,n}(t) = g(t-m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$

$g(t)$: Gabor atom (function), prototype pulse

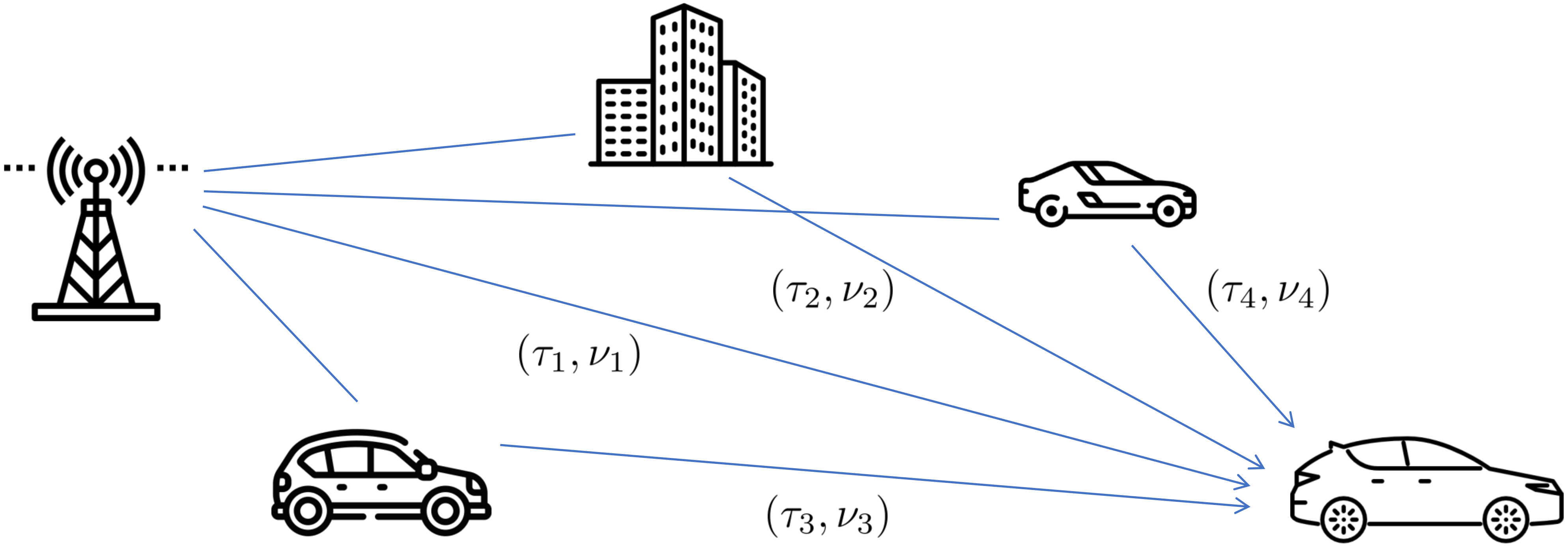
$$\hat{c}_{m,n} = \langle x, \gamma_{m,n}\rangle = \int x(t)\gamma^*_{m,n}(t)dt$$

$$\gamma_{m,n}(t) = \gamma(t-m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$

- WH sets: $(g,\mathcal{T},\mathcal{F}) = \{g_{m,n}(t)\}_{m,n\in\mathbb{Z}}$, $(\gamma,\mathcal{T},\mathcal{F}) = \{\gamma_{m,n}(t)\}_{m,n\in\mathbb{Z}}$
- WH frames: **Complete or overcomplete** WH sets with guaranteed numerical stability of reconstruction

JTFR	Sampling	Completeness	Frame for $(g,\frac{1}{\mathcal{F}},\frac{1}{\mathcal{T}}), (\gamma,\frac{1}{\mathcal{F}},\frac{1}{\mathcal{T}})$	(Bi)orthogonal WH sets exist?
$\mathcal{R}=\mathcal{T}\mathcal{F}>1$	Undercritical	Incomplete	✓ dual/tight	Yes
$\mathcal{R}=\mathcal{T}\mathcal{F}=1$	Critical	Complete	✓ dual/tight	Yes
$\mathcal{R}=\mathcal{T}\mathcal{F}<1$	Overcritical	Overcomplete	× dual/tight	No

Mobile Channel Models



- Doubly-selective channel with both time and frequency dispersion
- Deterministic model: delay-Doppler spread function, namely spreading function
- Path based model :

$$h(\tau,\nu) = \sum_{p=1}^{P} h_p \delta(\tau - \tau_p)\delta(\nu - \nu_p)$$

$$\tau_p = l_p \frac{1}{W_0}, \ \nu_p = k_p \frac{1}{T_0}, \ l_p, k_p \in \mathbb{Z}$$

Sampling rate : W_0 Duration : T_0

Orthogonal Delay-Doppler Division Multiplexing (ODDM) Modulation

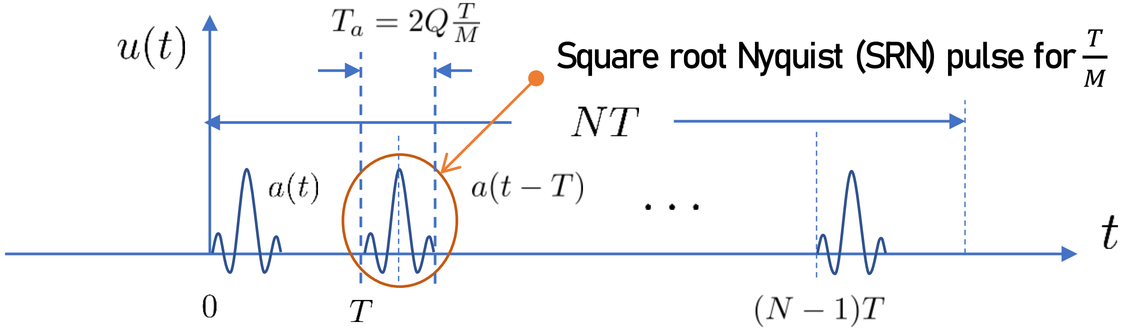
- Time resolution: $\mathcal{T} = \frac{1}{W_0}$, Frequency resolution: $\mathcal{F} = \frac{1}{T_0}$
- In ODDM, we have $W_0 = \frac{M}{T}$ and $T_0 = NT$, therefore $\mathcal{R} = \frac{1}{MN} \ll 1$.
- ODDM waveform

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X(m,n) u(t - m\frac{T}{M}) e^{j2\pi \frac{n}{NT}(t - m\frac{T}{M})}$$

Orthogonal pulses for $\mathcal{R} \ll 1$???

- DD Plane Orthogonal Pulse (DDOP)

$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T)$$



- Ambiguity function of $u(t)$

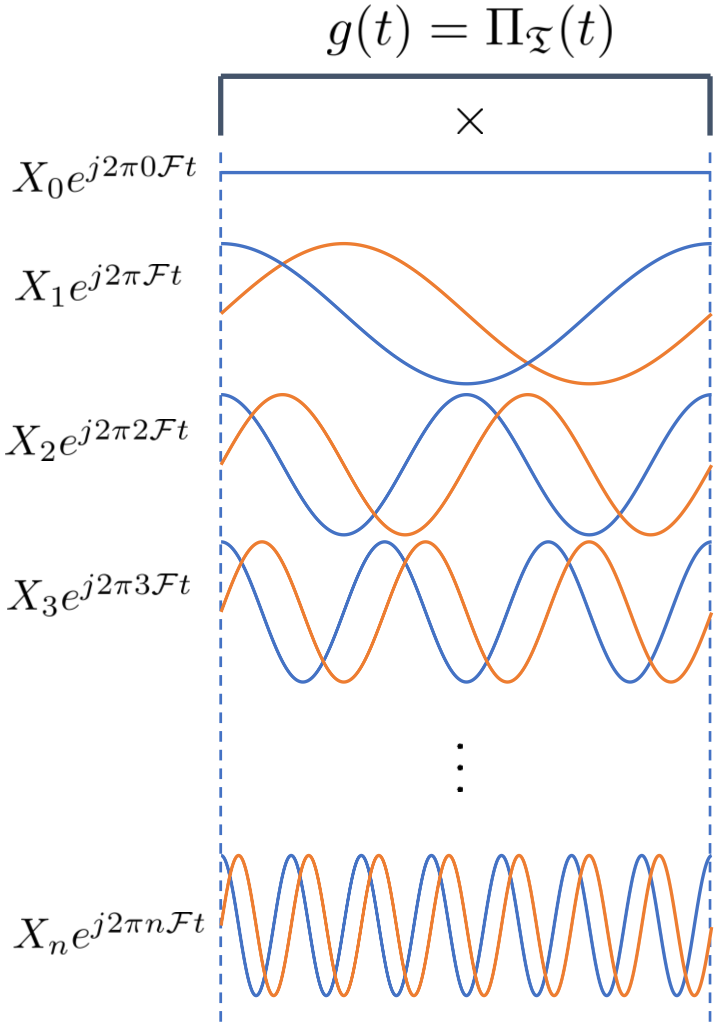
$$A_{u,u}\left(m\frac{T}{M}, n\frac{1}{NT}\right) = \delta(m)\delta(n), \forall |m| \le M-1, |n| \le N-1$$

Local (Bi)Orthogonality for WH Subsets

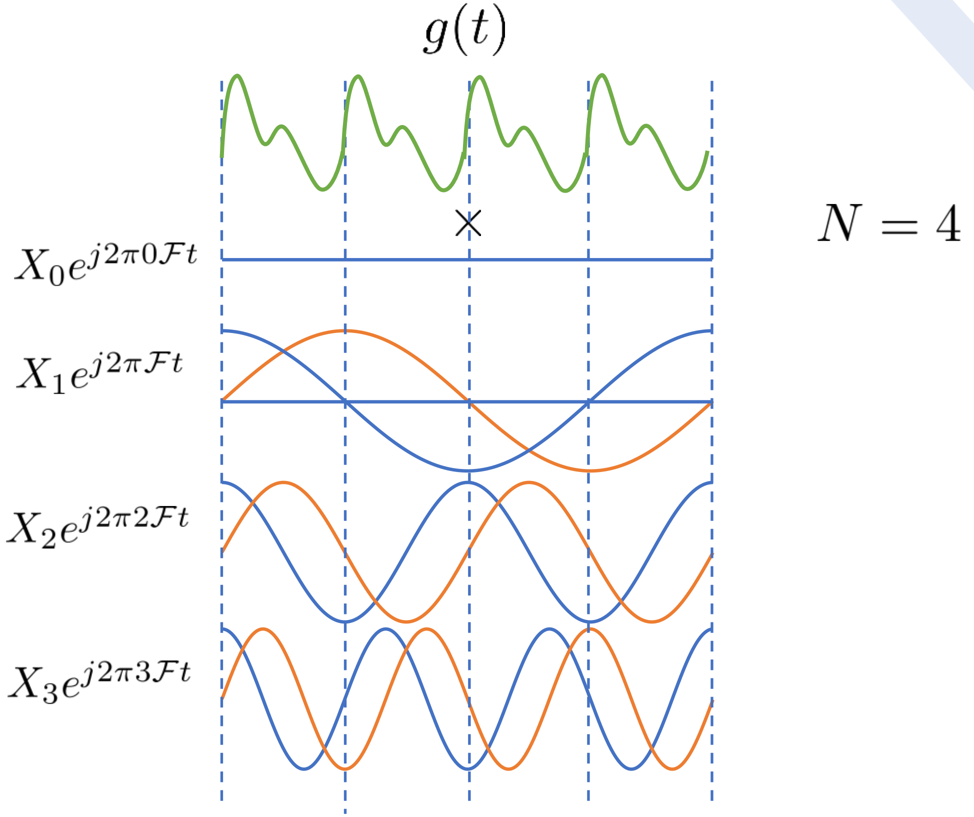
- WH sets: $(g, \mathcal{T}, \mathcal{F}) = \{g_{m,n}(t)\}_{m,n\in\mathbb{Z}}, \ (\gamma, \mathcal{T}, \mathcal{F}) = \{\gamma_{m,n}(t)\}_{m,n\in\mathbb{Z}}$
- WH subset: $(g, \mathcal{T}, \mathcal{F}, M, N) = \{g_{m,n}(t)\}_{0\le m\le M-1, 0\le n\le N-1}$
  $(\gamma, \mathcal{T}, \mathcal{F}, M, N) = \{\gamma_{m,n}(t)\}_{0\le m\le M-1, 0\le n\le N-1}$
- (Bi)Orthogonality among WH sets:
  - $m, n \in \mathbb{Z} \Rightarrow$ Global (bi)orthogonality governed by the WH frame theory
- (Bi)Orthogonality among WH subsets:
  - $0 \le m \le M-1, 0 \le n \le N-1 \ \Rightarrow$ Local (bi)orthogonality
  - Local (bi)orthogonality is not necessarily governed by the WH frame theory
  - Local (bi)orthogonality is enough for a modulation in the TF region of interest.

Orthogonality w.r.t Frequency Resolution

$\mathfrak{T} = \frac{1}{\mathcal{F}}$



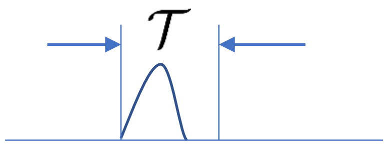
- When $n \in \mathbb{Z}$, $g(t)$ is the rectangular pulse with duration of $\mathfrak{T} = 1/\mathcal{F}$.



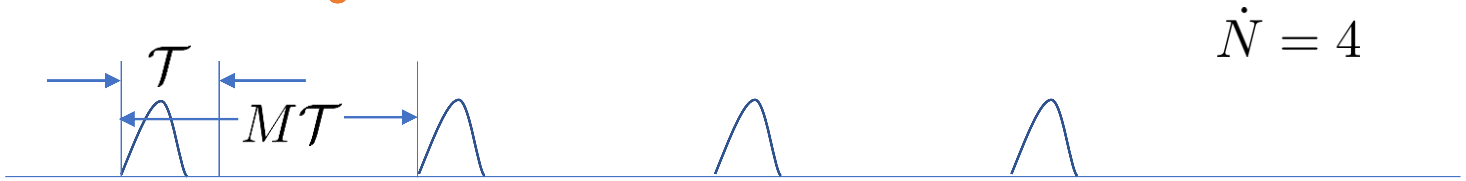
- When $|n| \leq N-1$, $g(t)$ can be any periodic function with a period of $\mathfrak{T}/N$ within the duration of $\mathfrak{T} = 1/\mathcal{F}$. (Lemma 1)

Orthogonality w.r.t Time Resolution

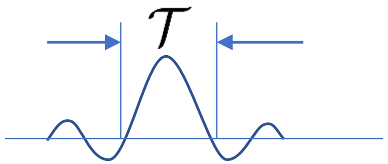
- When $m \in \mathbb{Z}$, $g(t)$ can be any function with duration $T_g \leq \mathcal{T}$, which is independent of M.



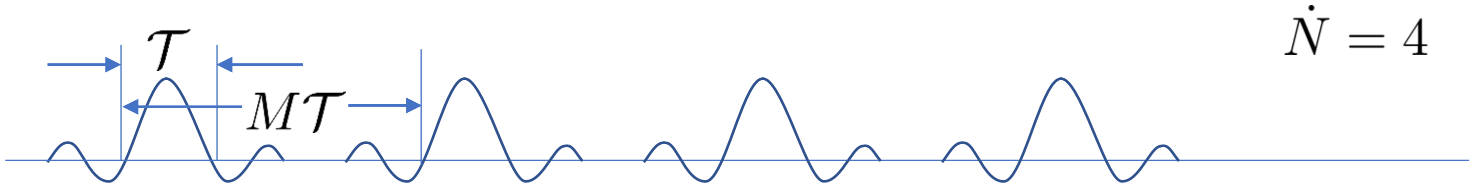
- When $|m| \leq M-1$, $g(t)$ consists of $\dot{N} > 1$ subpulses, each subpulse are temporally spaced by $M\mathcal{T}$ and each subpulse has a duration not longer than $\mathcal{T}$.



- When $n = 0$, another answer is
  - $m \in \mathbb{Z}$: SRN pulse for $\mathcal{T}$, whose duration is longer than $\mathcal{T}$

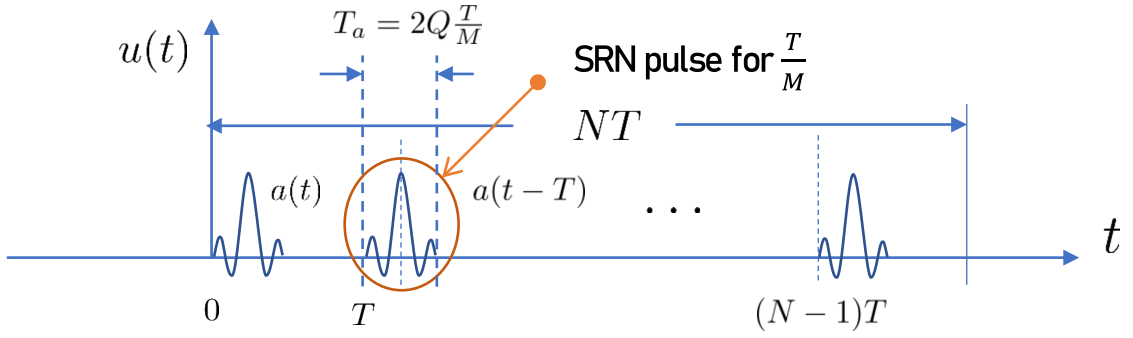


  - $|m| \leq M-1$: $g(t)$ consists of $\dot{N} > 1$ SRN subpulses, each subpulse are temporally spaced by $M\mathcal{T}$ and SRN subpulses can have any duration

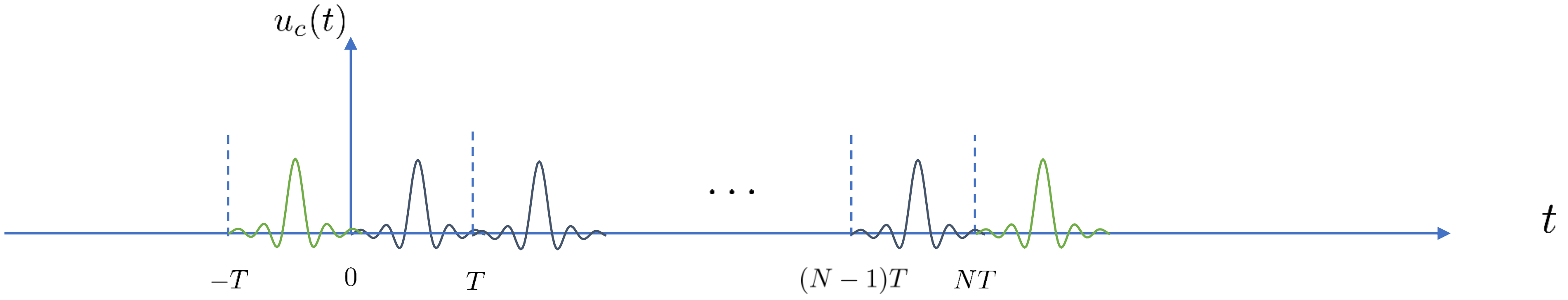


General DDOP Design

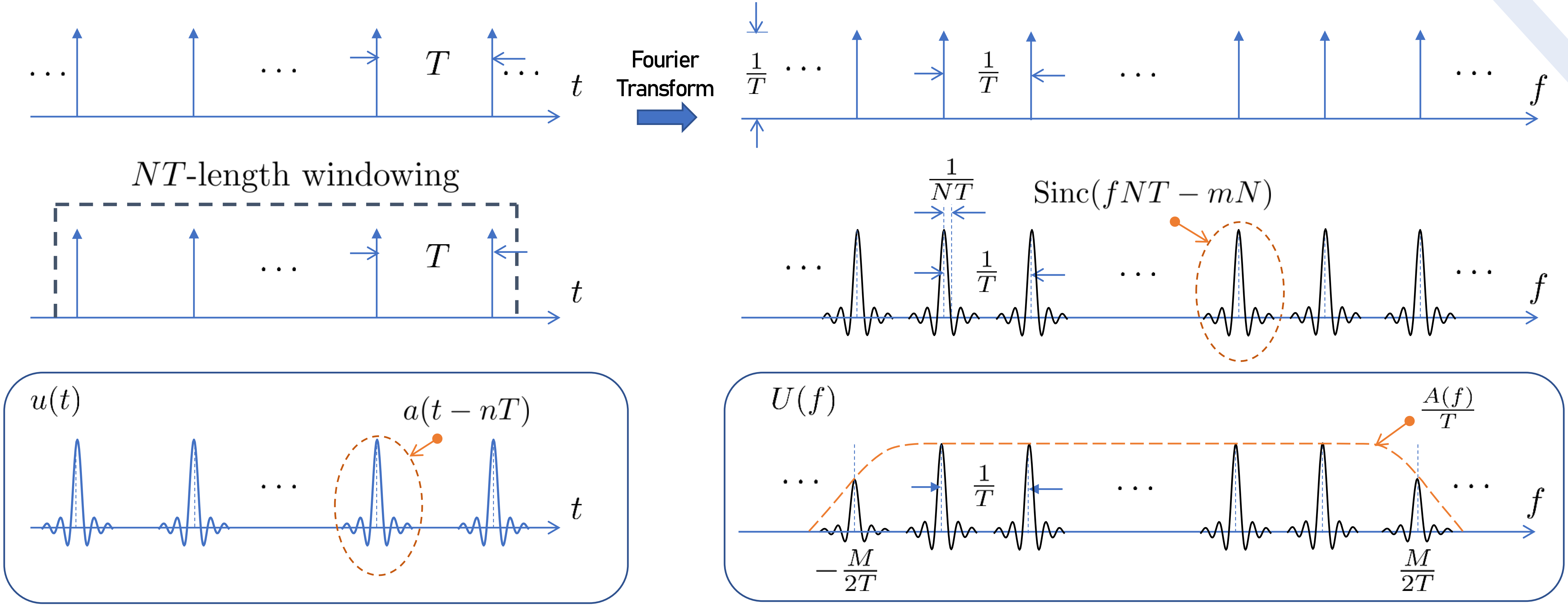
- Combine the aforementioned results , we obtain the DDOP
- In the original DDOP design, T_a, the duration of SRN pulse need to be $T_a \ll T$.



- The constraint of T_a can be relaxed by adding cyclic prefix and cyclic suffix, leading to a general DDOP (Lemma 2)



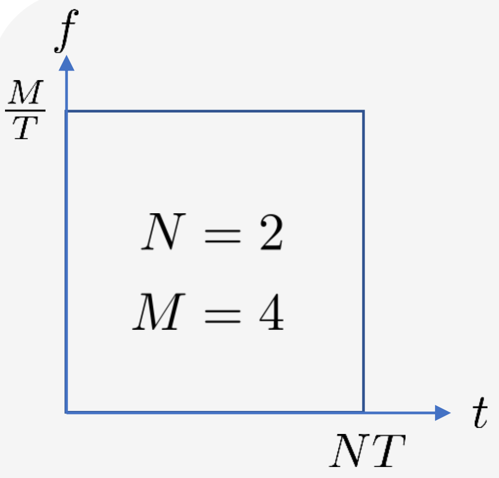
Frequency Domain Representation of DDOP



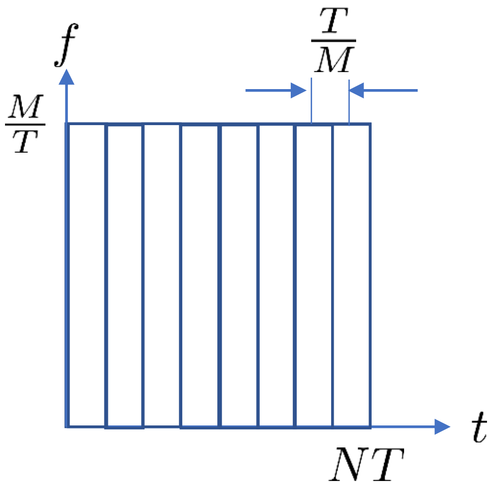
$$U(f) = \frac{e^{-j2\pi f\tilde{T}}}{T} A(f) \sum_{n=-\infty}^{\infty} e^{j2\pi \frac{n(N-1)}{2}} \mathrm{Sinc}(fNT - nN)$$

- Phase terms are ignored for the purpose of display

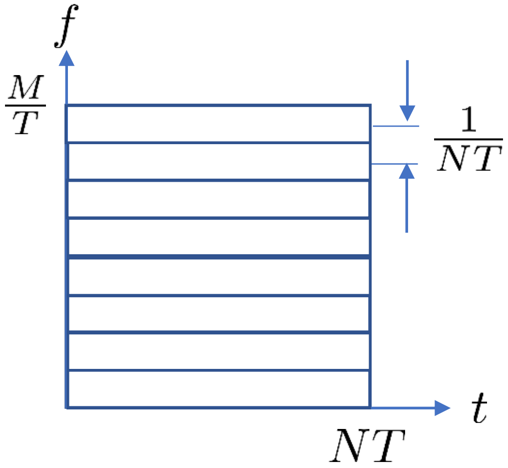
TF Signal Localization Comparison



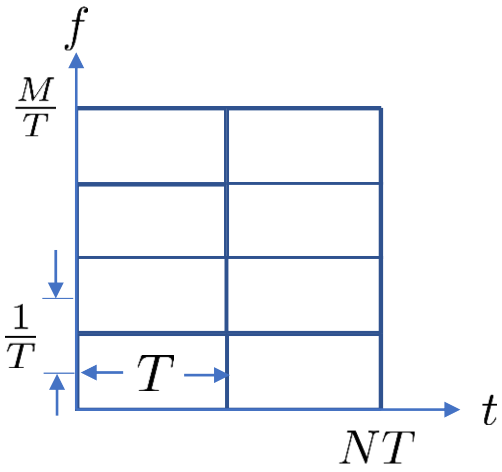
- Landau-Pollak Theorem
- DoF≈ $\frac{M}{T} \times NT = MN$
- MN symbols



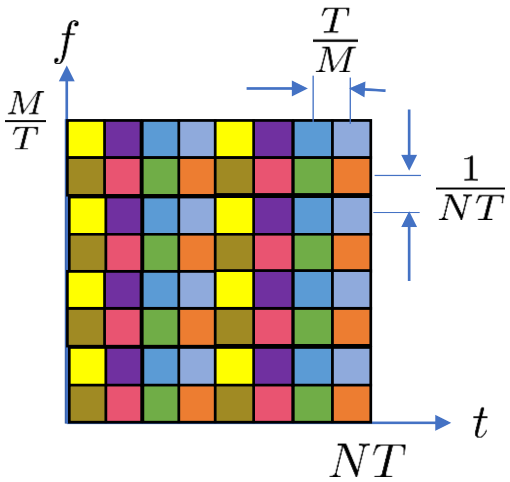
TDM(SC)
DoF: MN



FDM(OFDM)
DoF: MN



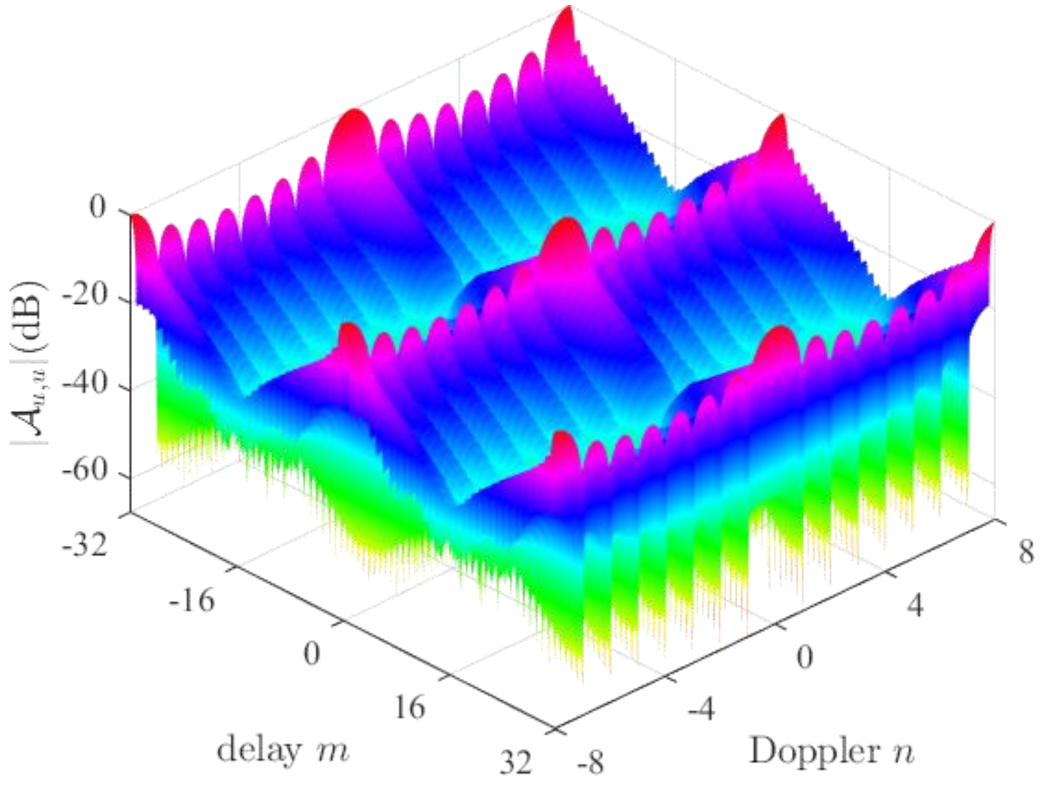
OFDM
DoF: MN



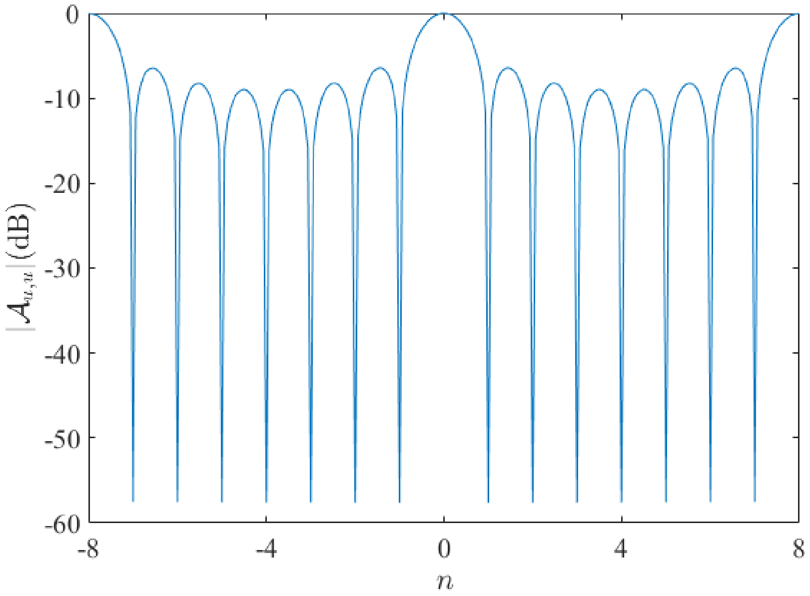
ODDM
DoF: MN

DDOP's Ambiguity Function

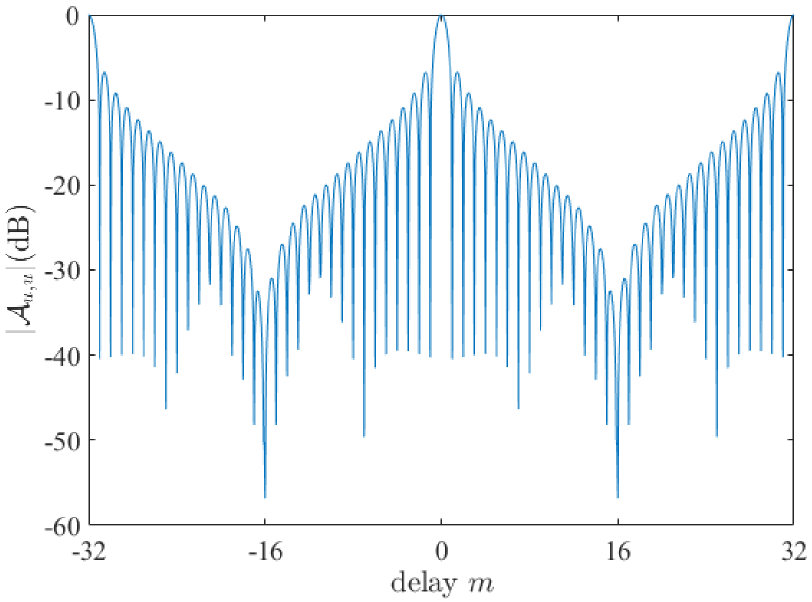
$$A_{u_c,u}\left(m\frac{T}{M}, n\frac{1}{NT}\right) = \delta(m)\delta(n), \forall |m| \leq M-1, |n| \leq N-1$$



$$M = 32, N = 8, \rho = 0.1$$



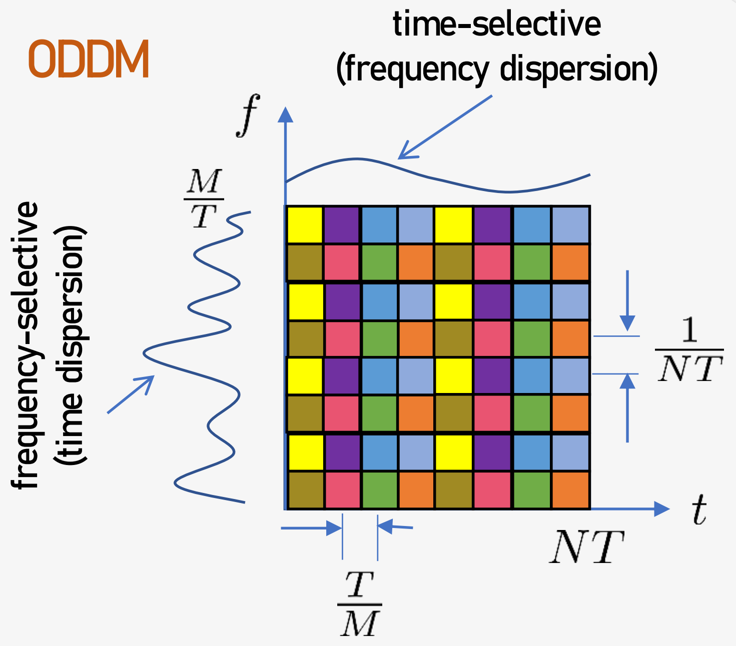
$m = 0$

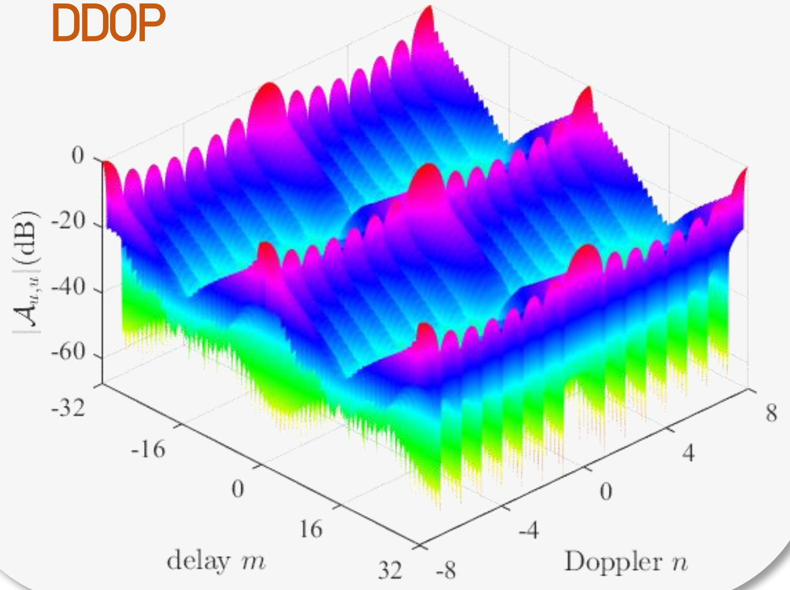


$n = 0$

Conclusion

- Justify the existence of DDOP
  - without violating the WH frame theory
  - local (bi)orthogonality
- Sufficient conditions for locally orthogonal pulses
- General DDOP design
- Frequency domain representation of DDOP
- TF signal localization comparison





- H. Lin and J. Yuan, "Multicarrier Modulation on Delay-Doppler Plane: Achieving Orthogonality with Fine Resolutions," IEEE ICC 2022.
- H. Lin and J. Yuan, "Orthogonal Delay-Doppler Division Multiplexing Modulation," IEEE Trans. Wireless Commun., Early access.