

Multicarrier Modulation on Delay-Doppler Plane: Achieving Orthogonality with Fine Resolutions

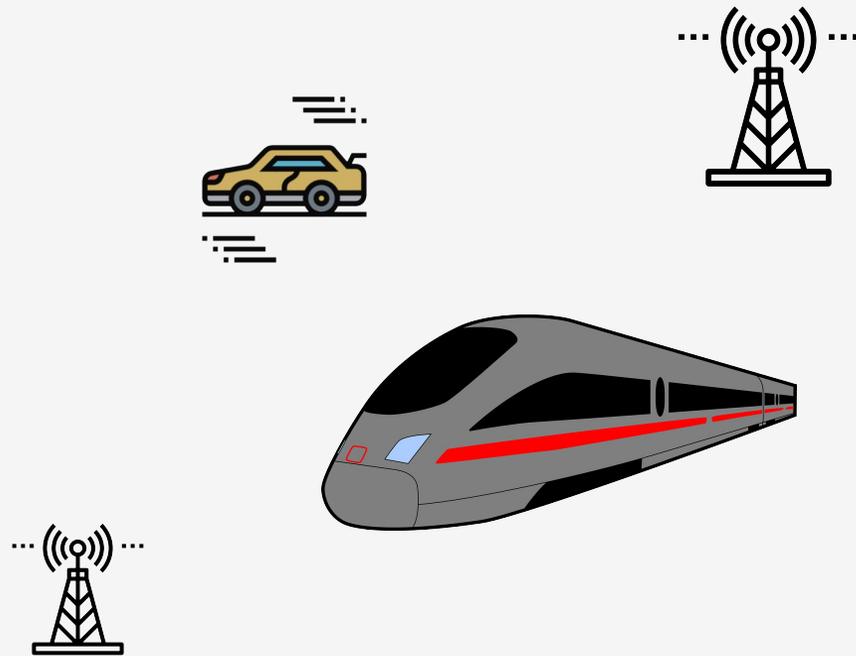
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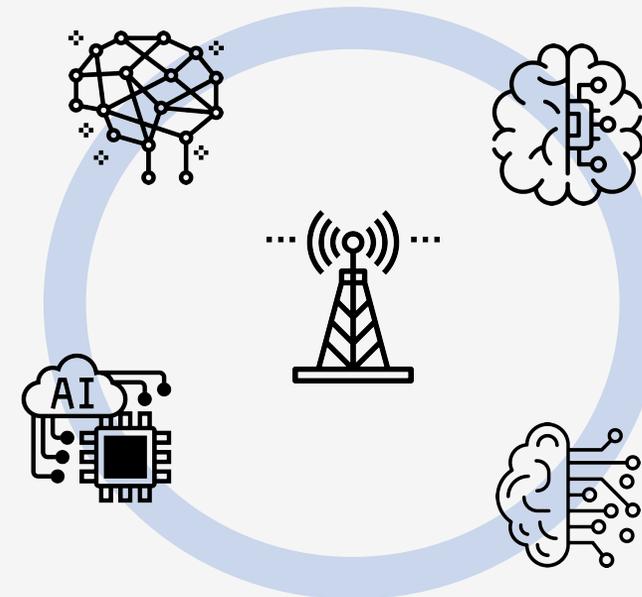
6G Scenarios

- High Reliability Communication



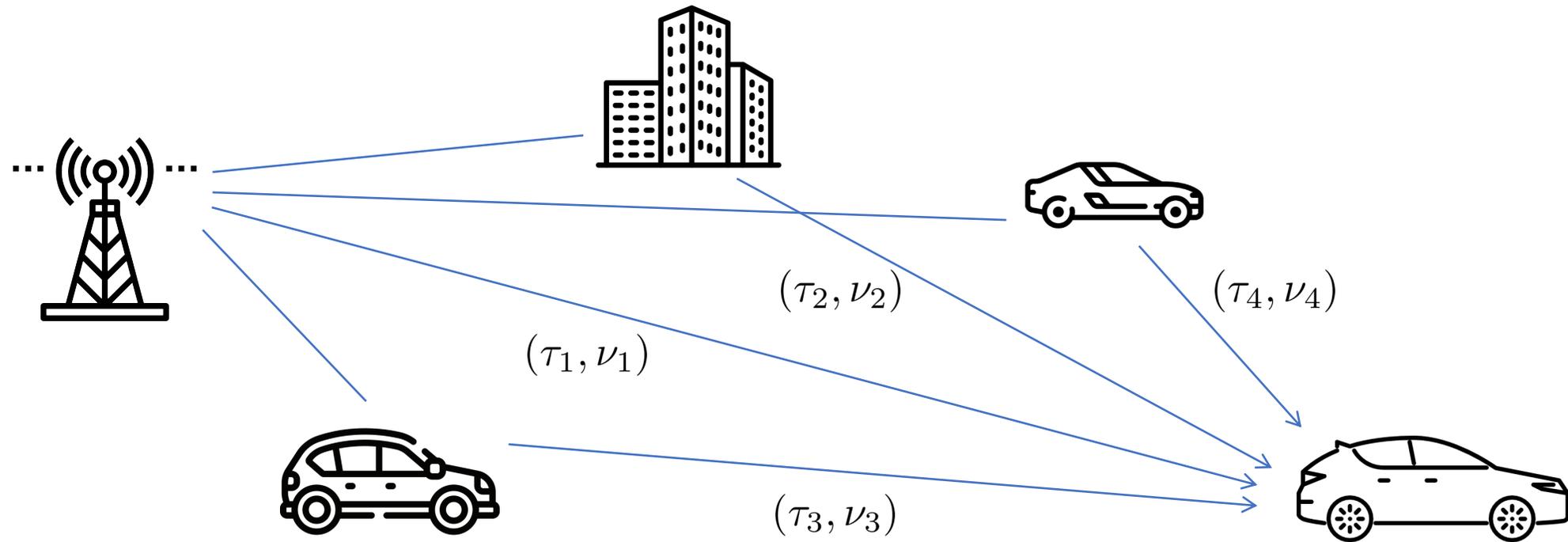
High mobility environment

- Connected Intelligence



Integrated sensing and communication (ISAC)

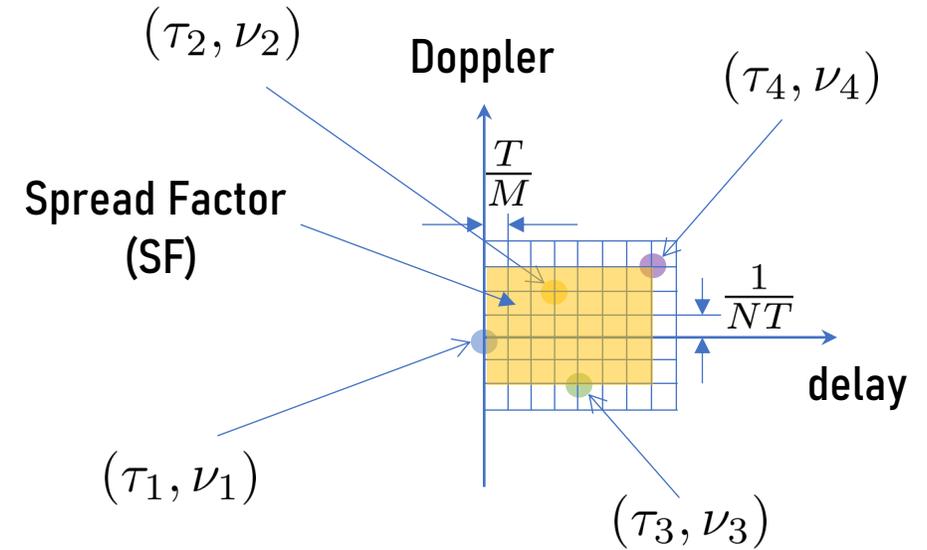
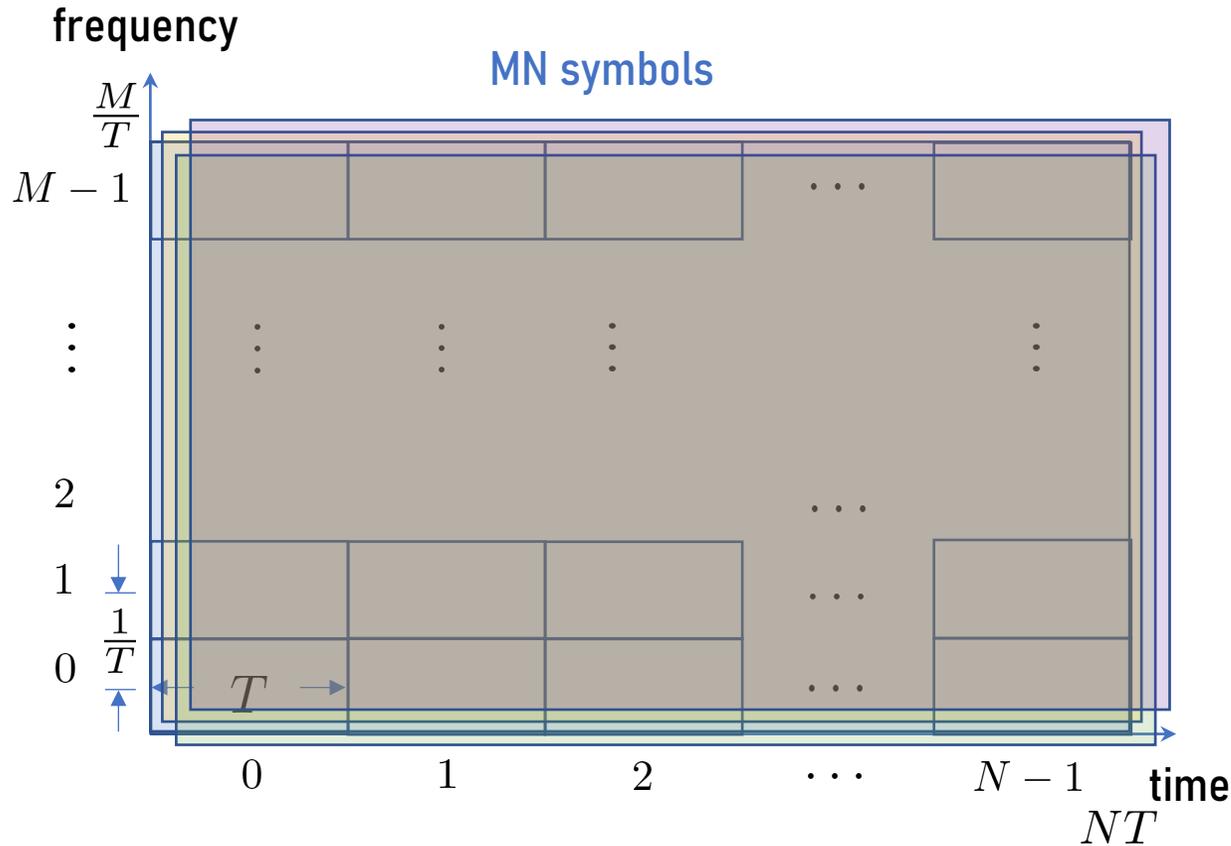
Mobile Channel Models



- Doubly-selective channel with both time and frequency dispersion
- Statistical models: WSSUS, Rayleigh, Rician, Nakagami-m
- **Deterministic model:** delay-Doppler spread function, namely spreading function $\mathcal{S}(\tau, \nu)$

➤ **Path based model:**
$$h(\tau, \nu) = \sum_{p=1}^P h_p \delta(\tau - \tau_p) \delta(\nu - \nu_p)$$

Time-Frequency (TF) and Delay-Doppler (DD) Planes



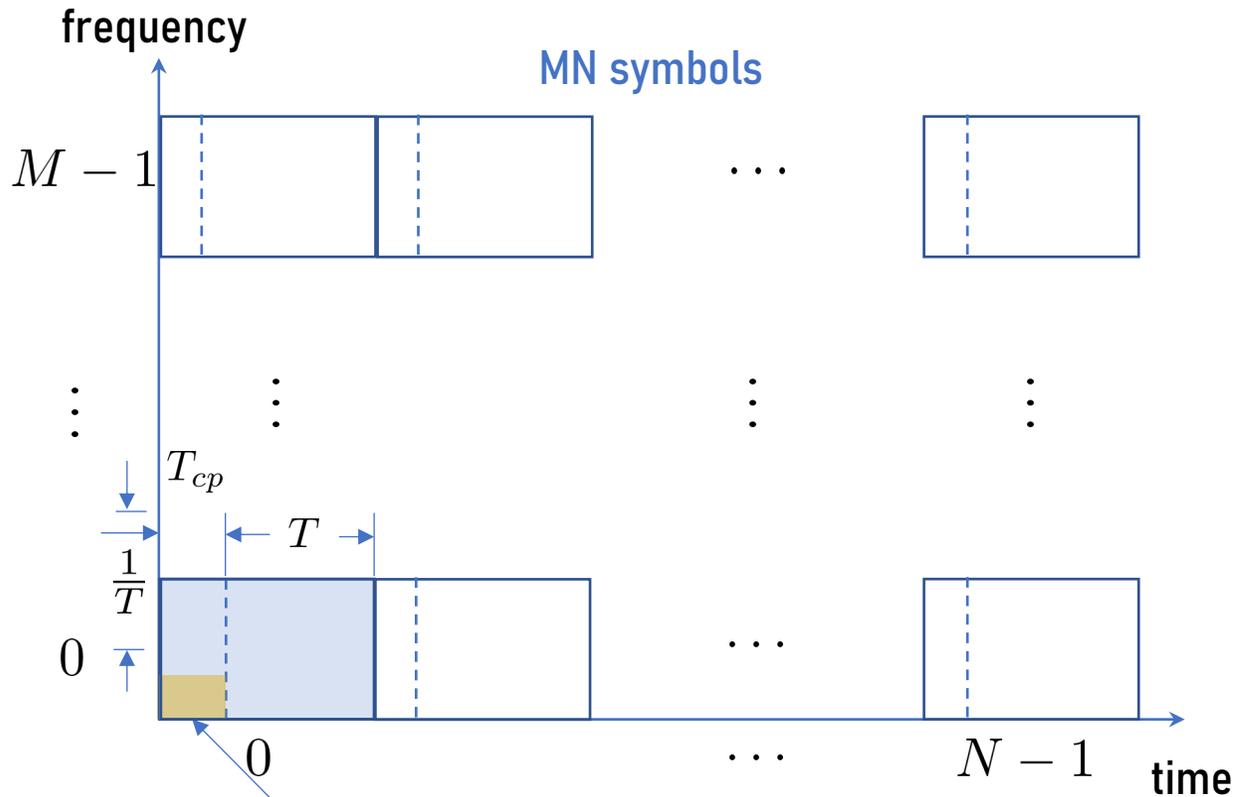
$$\tau_p = l_p \frac{T}{M}, \quad \nu_p = k_p \frac{1}{NT}, \quad l_p, k_p \in \mathbb{Z}$$

Sampling rate: $\frac{M}{T}$ Duration: NT

TF plane for modulation \longleftrightarrow TF resolution mismatch DD plane for channel

Complicated ISI and ICI (off-the-grid)

Delay and Doppler in 4G/5G OFDM

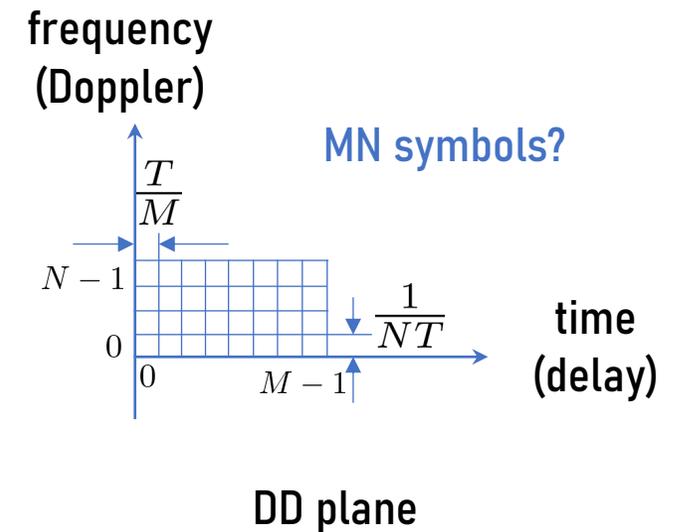


$SF < 1 \Rightarrow$ Underspread

- 4G/5G OFDM : **TF plane** MC modulation
- Strategy : Avoid the DD-induced ISI and ICI
- Approach : Introduce redundancy based on SF
- CP per symbol and/or large subcarrier spacing \Rightarrow low spectral efficiency
- Low signal resolution \Rightarrow Sensing and communication are handled separately

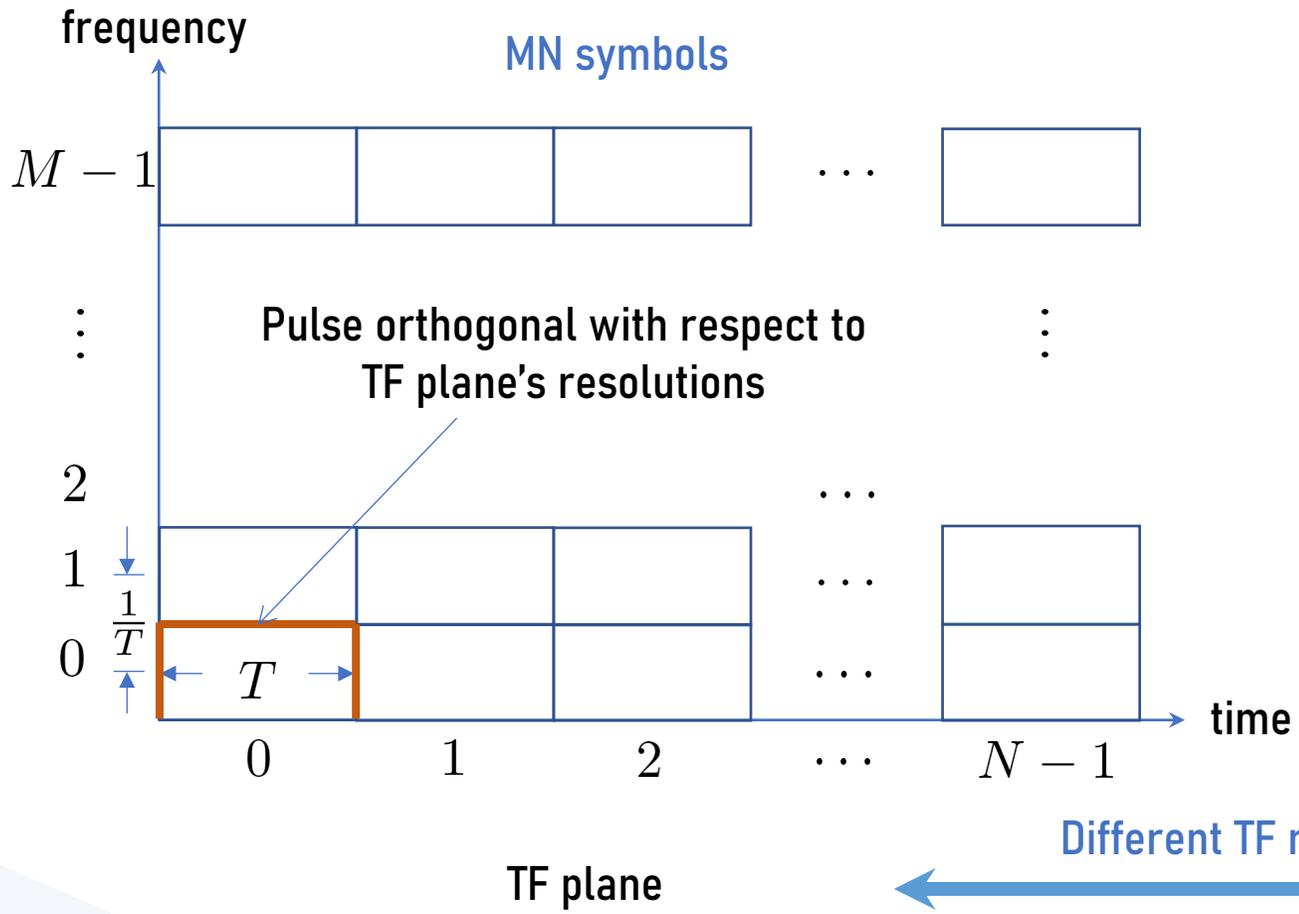
A Candidate Waveform for 6G

- 6G : **DD plane** MC modulation?
- Strategy : Harvest path-induced TF diversity
- Approach : Match signal and channel's resolution
- On-the-gird ISI and ICI (low-complexity equalization)
- TF diversity \Rightarrow High reliability communication
- Minimum redundancy \Rightarrow High spectral efficiency
- High signal resolution \Rightarrow ISAC

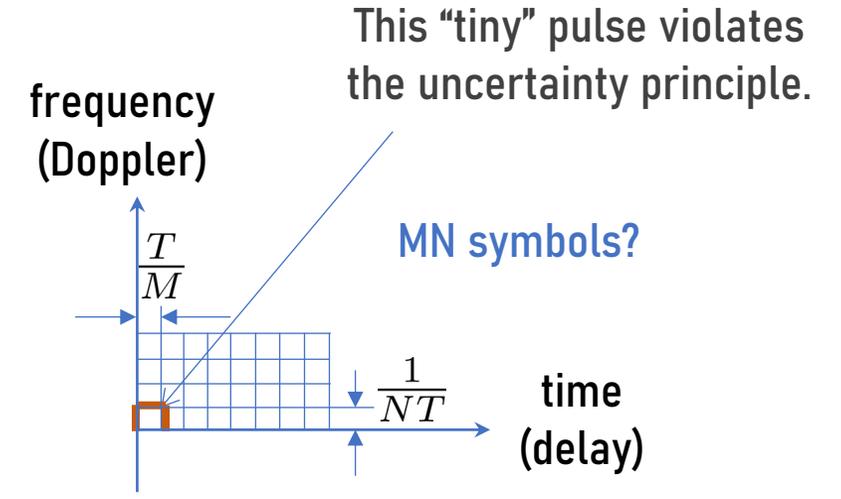


Fundamental Issue of DD Plane MC Modulation

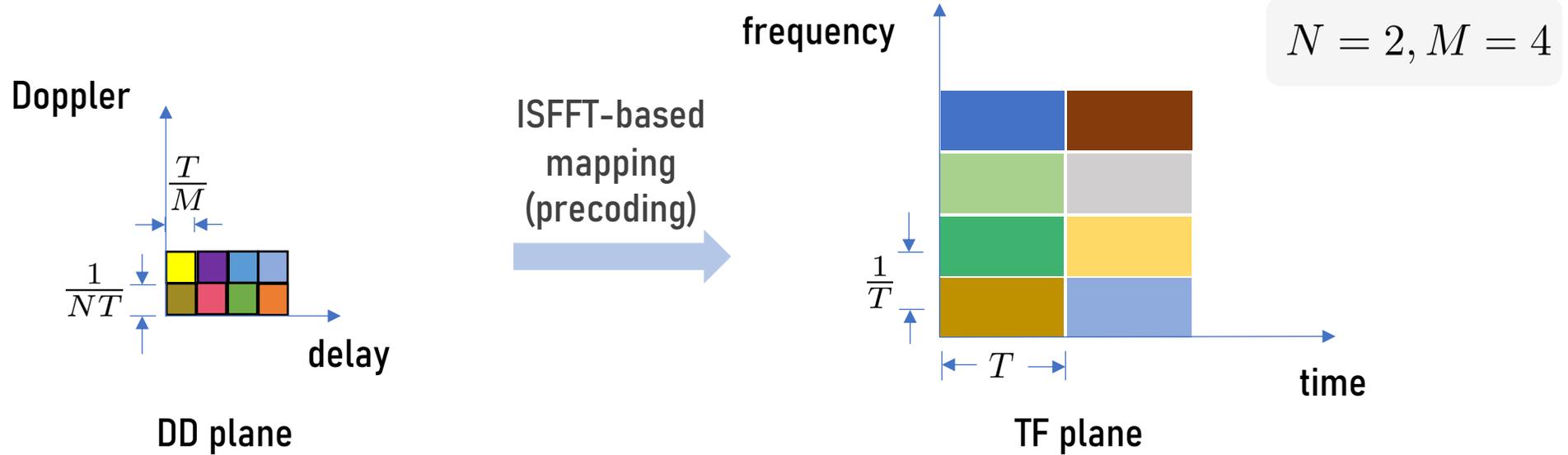
➤ Principle of modulation : One pulse for one symbol



No DD plane orthogonal pulse?

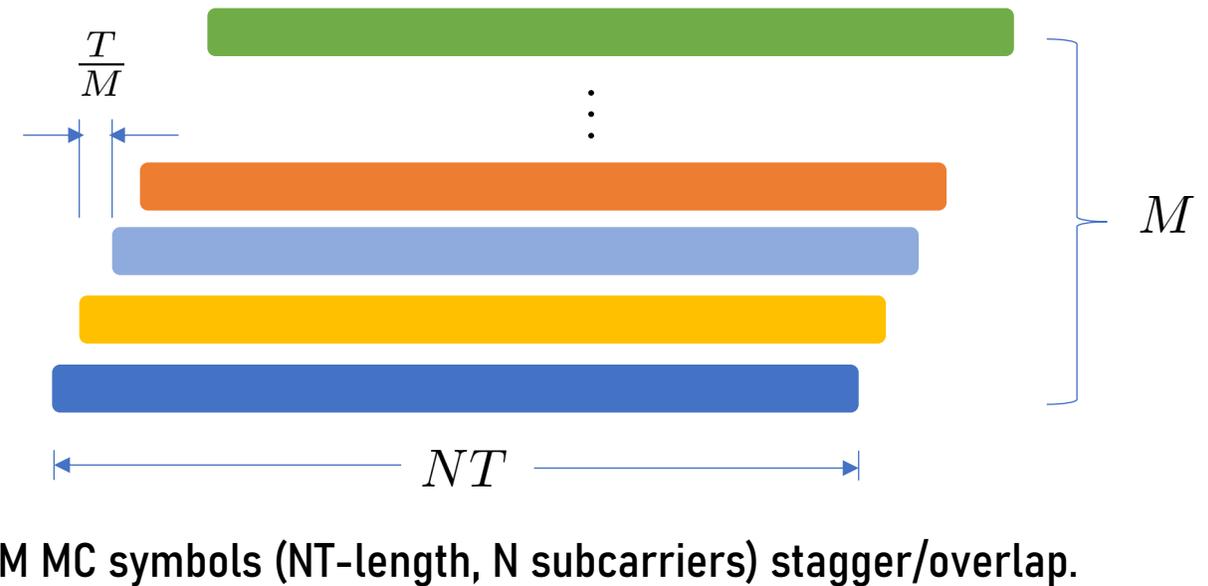
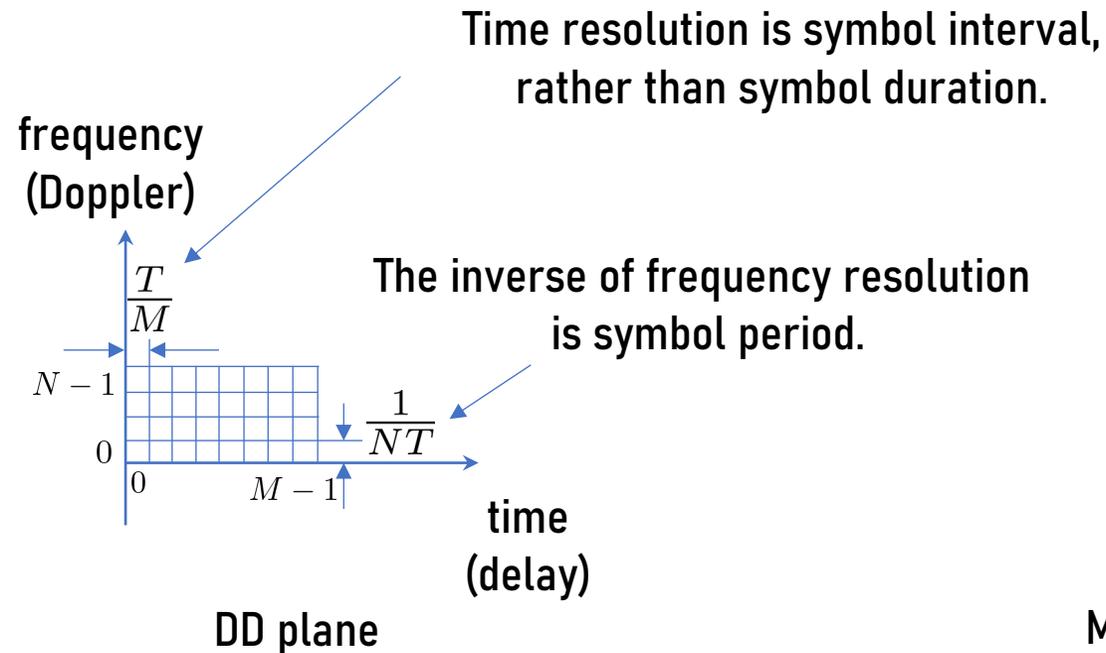


Orthogonal Time Frequency Space (OTFS) Modulation



- Maps signals from DD plane to TF plane, then use OFDM
- OTFS waveform is still orthogonal with respect to TF plane's resolutions
- OTFS's ideal pulse is said to satisfy **biorthogonal robust property**, however cannot be realized in practice.
- Suffers high OOB and complicated ISI and ICI

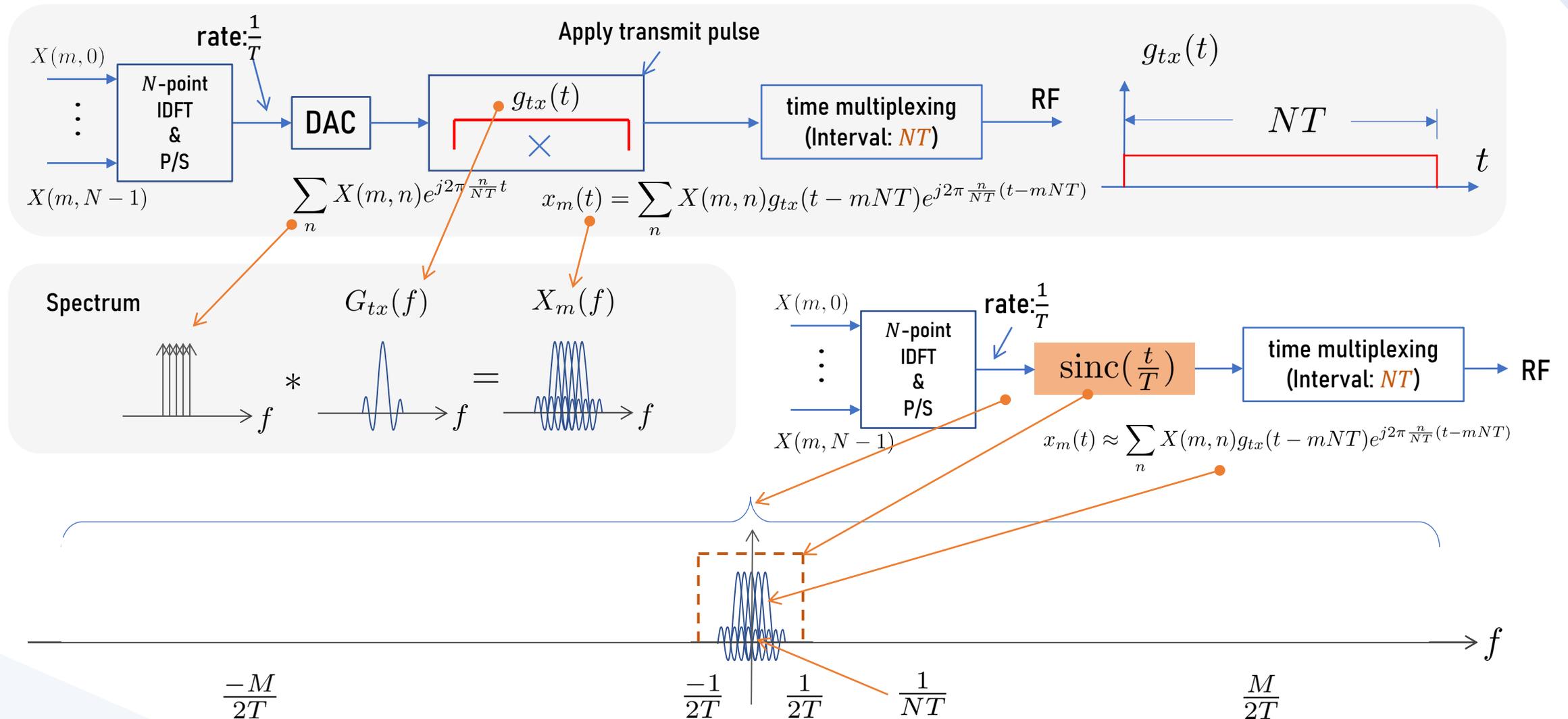
DD Plane MC Modulation



- DD plane MC modulation \Rightarrow A type of **staggered multitone (SMT) modulation**
- Long symbol period \Rightarrow Narrowband, Short symbol interval \Rightarrow Wideband. **How is this possible?**

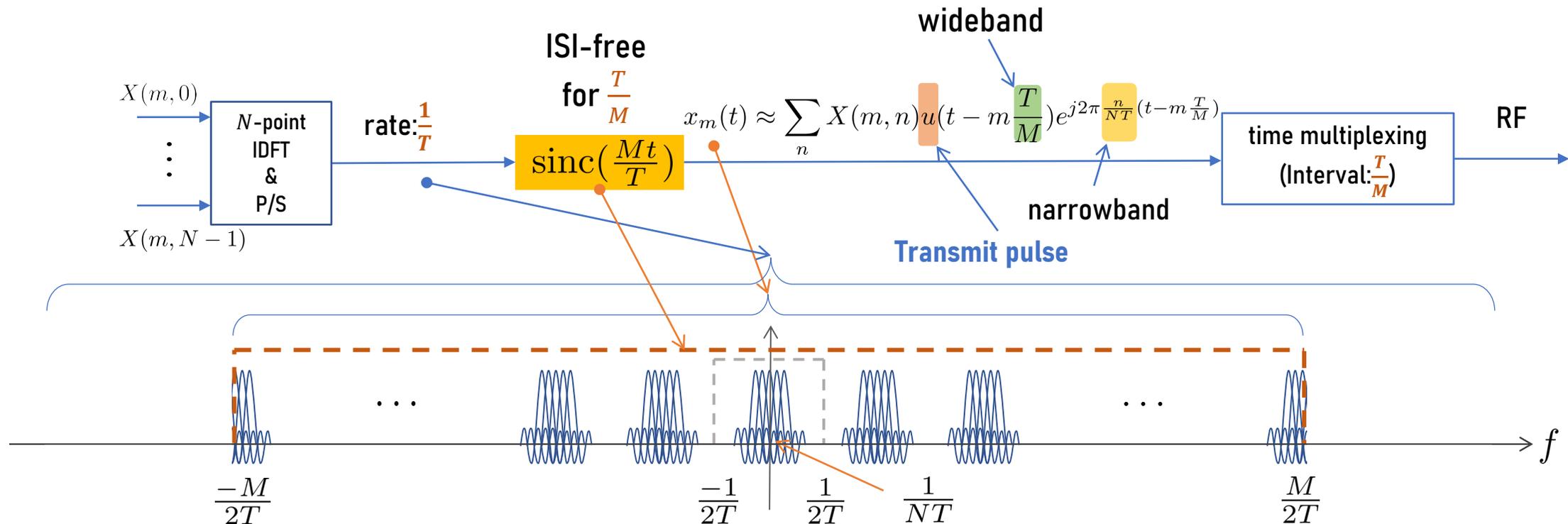
Revisit OFDM Pulse Shaping

- CP-free OFDM: **Symbol interval = Symbol period NT**



Pulse Shaping for DD Plane MC modulation

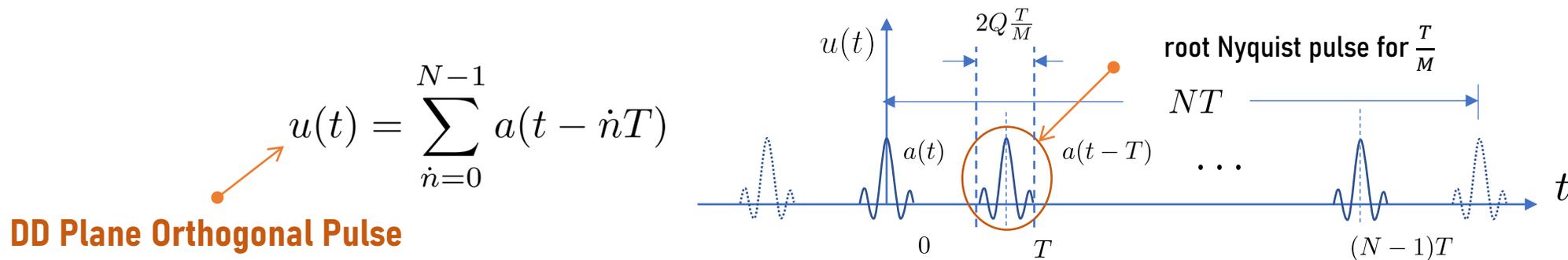
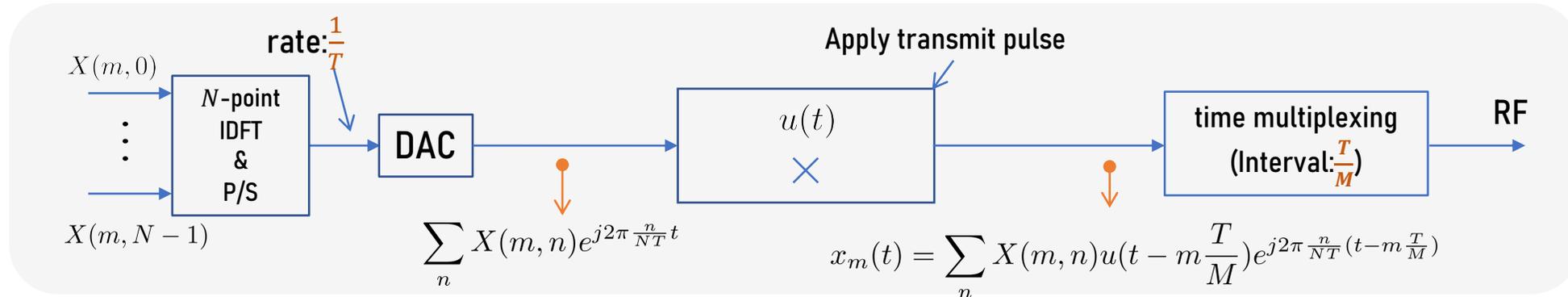
- Symbol interval $\frac{T}{M} \ll$ Symbol period NT



- Like SC modulation, use root Nyquist pulse for symbol interval $\frac{T}{M}$ (wideband)
- Keep the aliasing parts of signal to form **CP and cyclic suffix in frequency domain**

Transmit Pulse for DD Plane MC Modulation

- Symbol interval $\frac{T}{M} \ll$ Symbol period NT

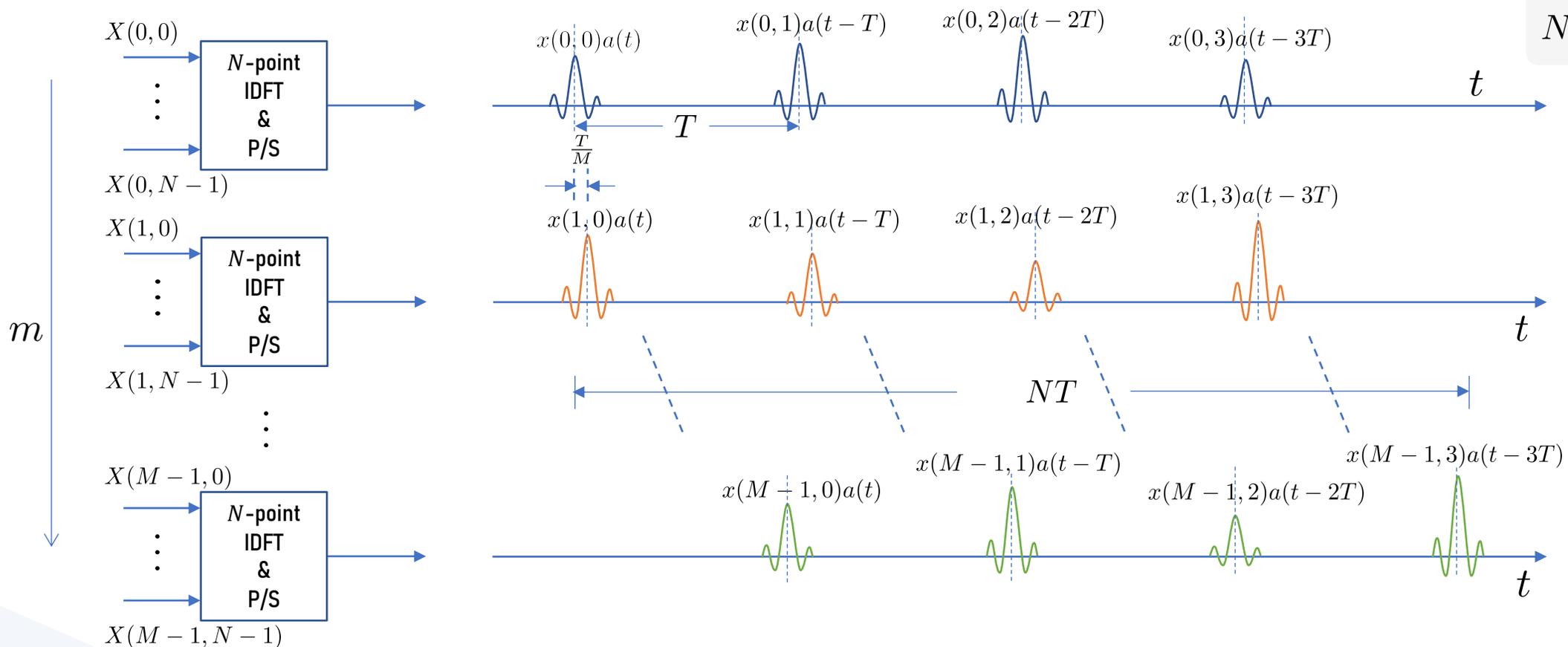


- Ambiguity function of $u(t)$

$$A_{u,u} \left(m \frac{T}{M}, n \frac{1}{NT} \right) = \delta(m) \delta(n), \forall |m| \leq M-1, |n| \leq N-1$$

Orthogonal Delay-Doppler Division Multiplexing (ODDM) Modulation

- ODDM is pulse-shaped by $u(t)$ and therefore orthogonal with respect to the DD plane's resolutions $\frac{T}{M}$ and $\frac{1}{NT}$.
- When $2Q \ll M$, the combination of N -point IDFT and $a(t)$ -based filtering **approximates** ODDM waveform.

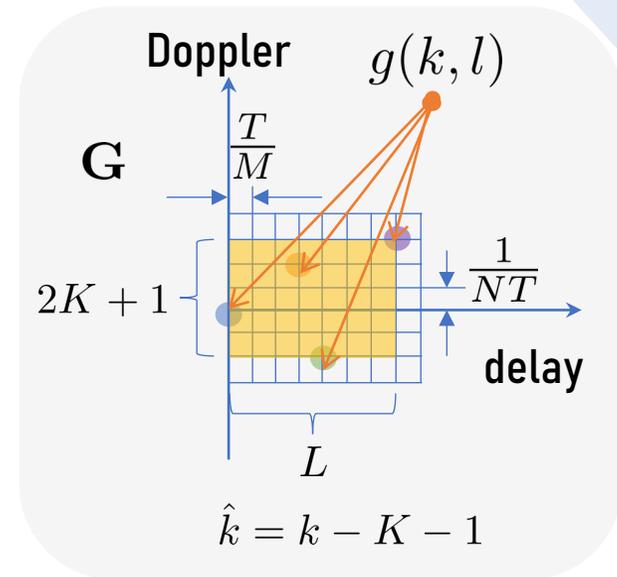


- Add a frame-wise CP to keep the periodicity

DD Domain Input-Output Relation

- Receive pulse (matched filter) : $u(t - m\frac{T}{M})e^{-j2\pi\frac{n}{NT}(t - m\frac{T}{M})}$
- Path's delay is integer multiples of $\frac{T}{M}$
- Path's Doppler is integer multiples of $\frac{1}{NT}$
- OFDM with integer timing/frequency offset
- See from the n th subcarrier of the m th symbol

$g(k, l)$: gain g corresponds to on-the-grid ISI (ICI) from the $[n - \hat{k}]_N$ th subcarrier of the $[m - l]_M$ th symbol



$$\begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{M-1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0^0 & & & \mathbf{H}_{L-1}^0 \mathbf{D} & \cdots & \cdots & \mathbf{H}_1^0 \mathbf{D} \\ \vdots & \ddots & & \vdots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{H}_{L-2}^{L-2} & \cdots & \cdots & \mathbf{H}_0^{L-2} & & & \mathbf{H}_{L-1}^{L-2} \mathbf{D} \\ \mathbf{H}_{L-1}^{L-1} & \cdots & \cdots & \mathbf{H}_0^{L-1} & & & \mathbf{H}_{L-1}^{L-1} \mathbf{D} \\ \vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & & & \mathbf{0} & & & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & & & \mathbf{H}_{L-1}^{M-1} & \cdots & \cdots & \mathbf{H}_0^{M-1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{M-1} \end{bmatrix} + \begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_{M-1} \end{bmatrix}$$

$$\mathbf{H}_l^m = \sum_{k=1}^{2K+1} g(k, l) e^{j2\pi\frac{\hat{k}(m-l)}{MN}} \mathbf{C}^{\hat{k}}$$

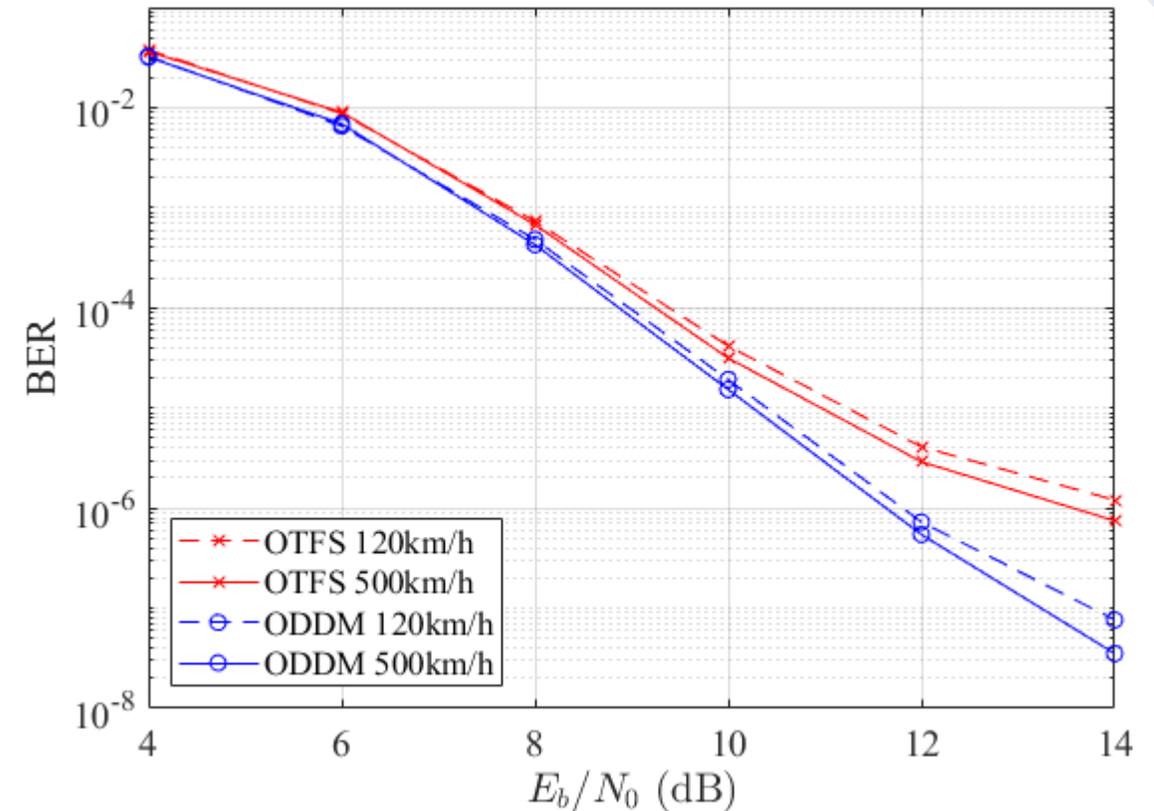
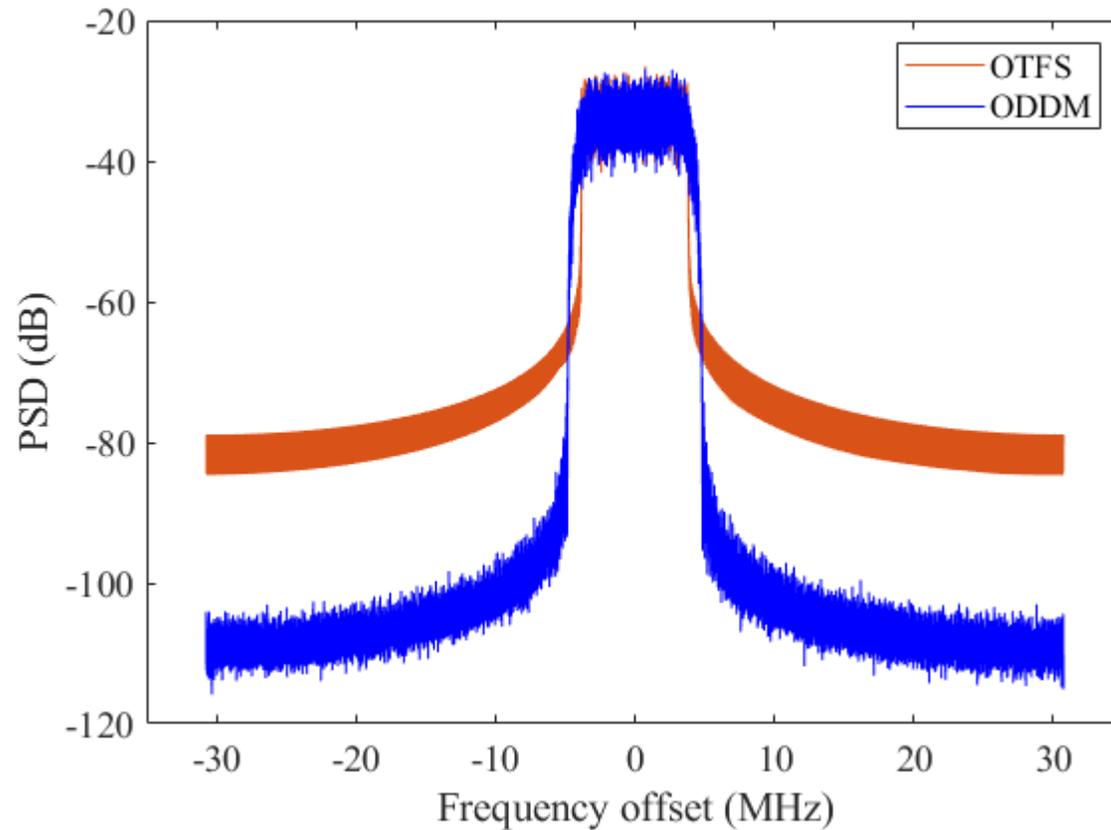
$$\mathbf{D} = \text{diag} \left\{ 1, e^{-j\frac{2\pi}{N}}, \dots, e^{-j\frac{2\pi(N-1)}{N}} \right\}$$

\mathbf{C} : $N \times N$ cyclic permutation matrix

- Equalization : MP, MMSE, DFE, ...

block-circulant-like structure brought by
time and frequency domain CP

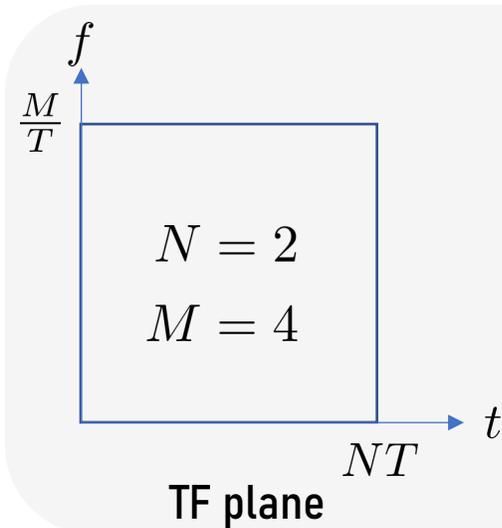
Simulation Results



➤ $M = 512$, $N = 64$, $\frac{1}{T} = 15\text{kHz}$, $f_c = 5\text{GHz}$, EVA Channel

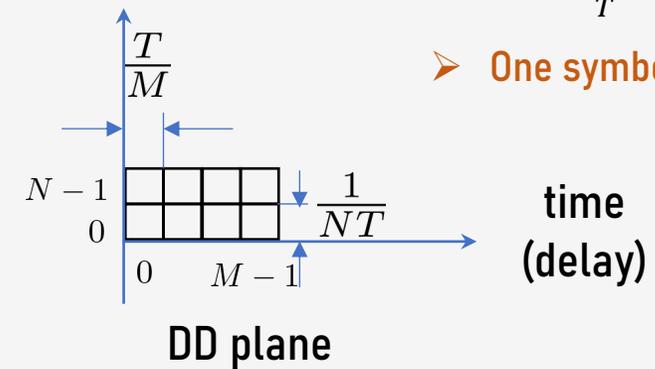
➤ $Q = 16$, roll-off factor = 0.25, Uncoded 4-QAM, MP Equalization

From the Viewpoint of DoF

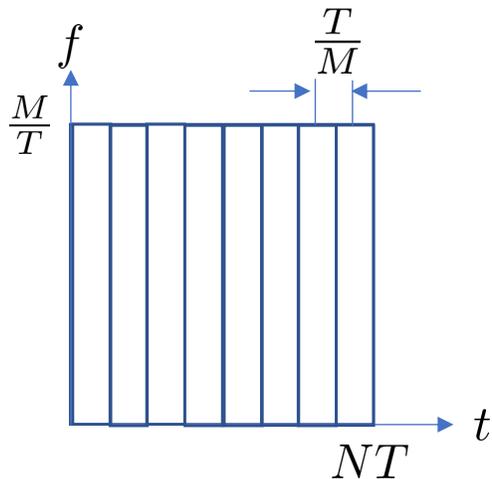


- Landau-Pollak Theorem
- $\text{DoF} \approx \frac{M}{T} \times NT = MN$
- MN symbols

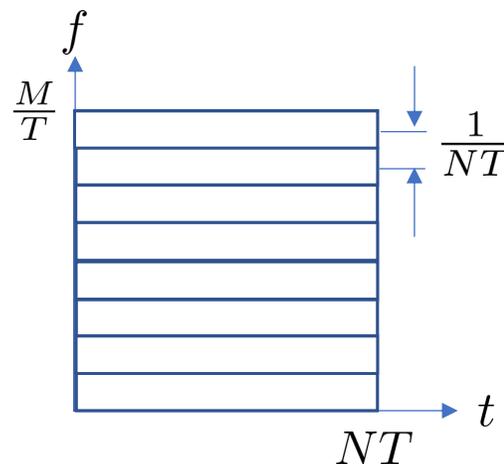
frequency
(Doppler)



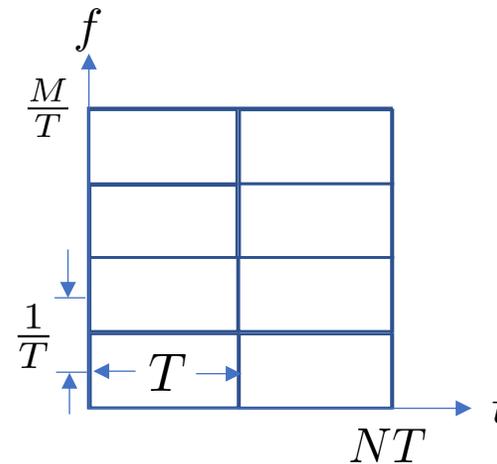
- $\text{DoF} \approx \frac{1}{T} \times T = 1$
- One symbol!



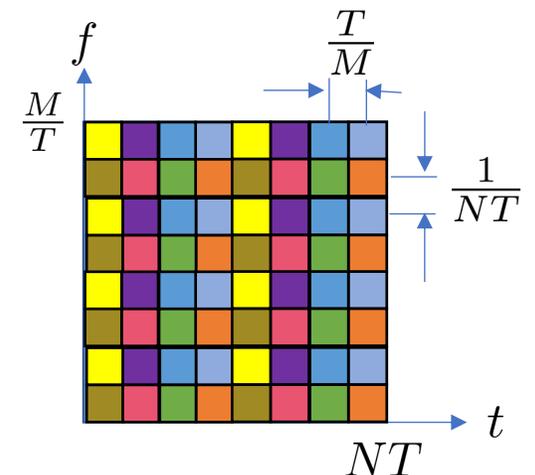
DoF: MN



DoF: MN

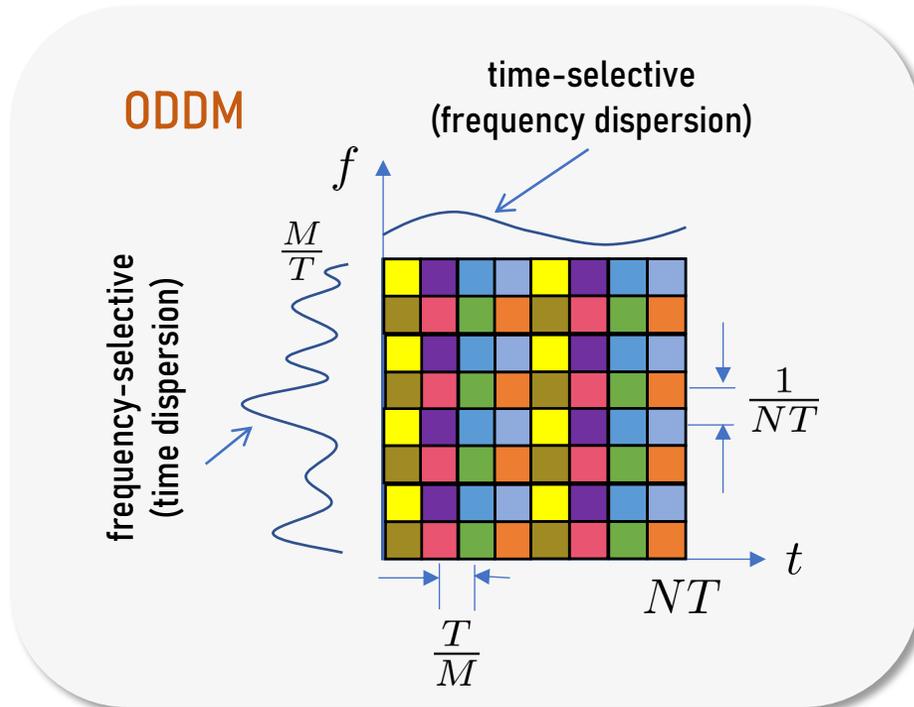


DoF: MN



DoF: MN

Conclusion



- A Novel Multicarrier Modulation Waveform
- Based on a newly discovered orthogonal pulse
- Orthogonal MC with respect to DD resolution
- A hybrid of TDM and FDM
- Coupling between signal and channel
- Low OOB, Low PAPR
- Flexible bandwidth, high spectral efficiency
- For high reliability communication
- For ISAC in future mobile communications

Thank you for your attention!