Orthogonal Delay-Doppler Division Multiplexing Modulation : A Novel Delay-Doppler Multi-Carrier Waveform

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Outline

1. Introduction
2. Wireless Channels with Delay-Doppler (DD) Domain Representation
3. OTFS Modulation
4. DD Plane Multi-Carrier (DDMC) Modulation
5. Results and Discussions

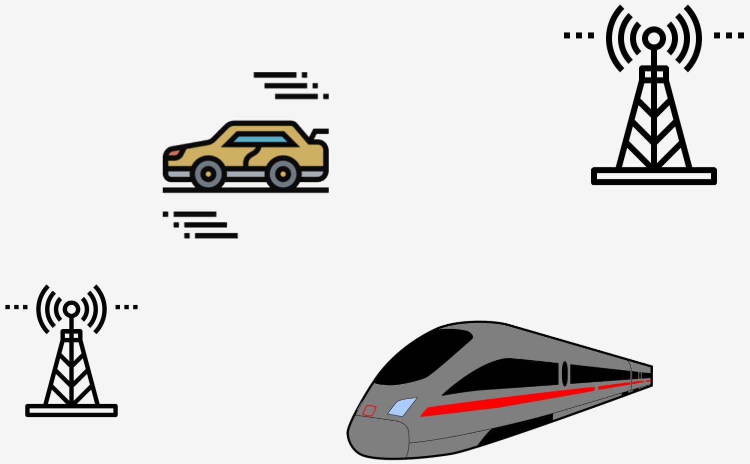
Introduction

Cellular Evolution	1G (1980's)	2G (1990's)	3G (2000's)	4G (2010's)	5G (2020's)
Waveform	Analog FDMA	TDMA, CDMA	CDMA	OFDM	OFDM
Data Rate	2.4 kbps	64 kbps	100 kbps - 56 Mbps	Up to 1 Gbps	> 1 Gbps
Carrier Frequency	800-900 MHz	850-1900MHz	1.6-2.5GHz	2-8 GHz	Sub- 6GHz, mmWave

- Design TX waveforms to support high rate, deal with fading and interference
  - TDMA
  - CDMA, MC-CDMA
  - OFDMA, MIMO-OFDMA
- Combating channel fading (1G, 2G, 3G) to exploiting channel fading (4G, 5G)

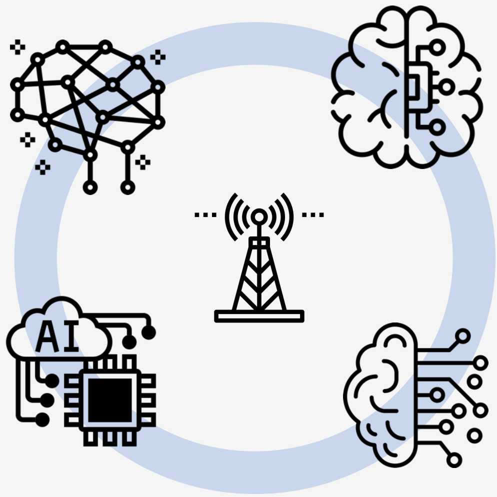
Introduction-6G Scenarios

- High Mobility



High Reliability Communication (HRC)

- Connected Intelligence



Integrated Sensing and Communication (ISAC)

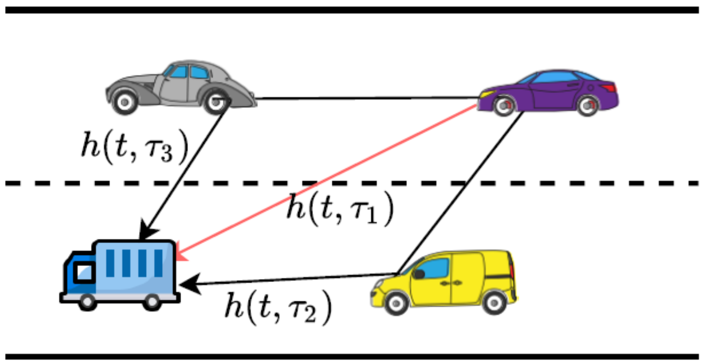
Is there any signal waveform better interacting with wireless channels?

- Robust and reliable against to distortion/impairments of doubly-selective channels?
- Viable choice for ISAC over doubly-selective channels?

Introduction-OTFS Modulation

- A new two-dimension modulation technique, compatible with 4G/5G OFDM
- Works well in high Doppler fast fading wireless channels
- Key idea: exploit underlying property: compact and sparse channel property in DD domain.
- Carries information in DD domain, couple with DD channel
- Aims to have minimal cross-interference as well as full diversity in DD domain

Doubly-selective



High-mobility environment

- R. Hadani, S. Rakib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch, and R. Calderbank, "Orthogonal time frequency space modulation," in Proc. IEEE WCNC, 2017, pp. 1–6

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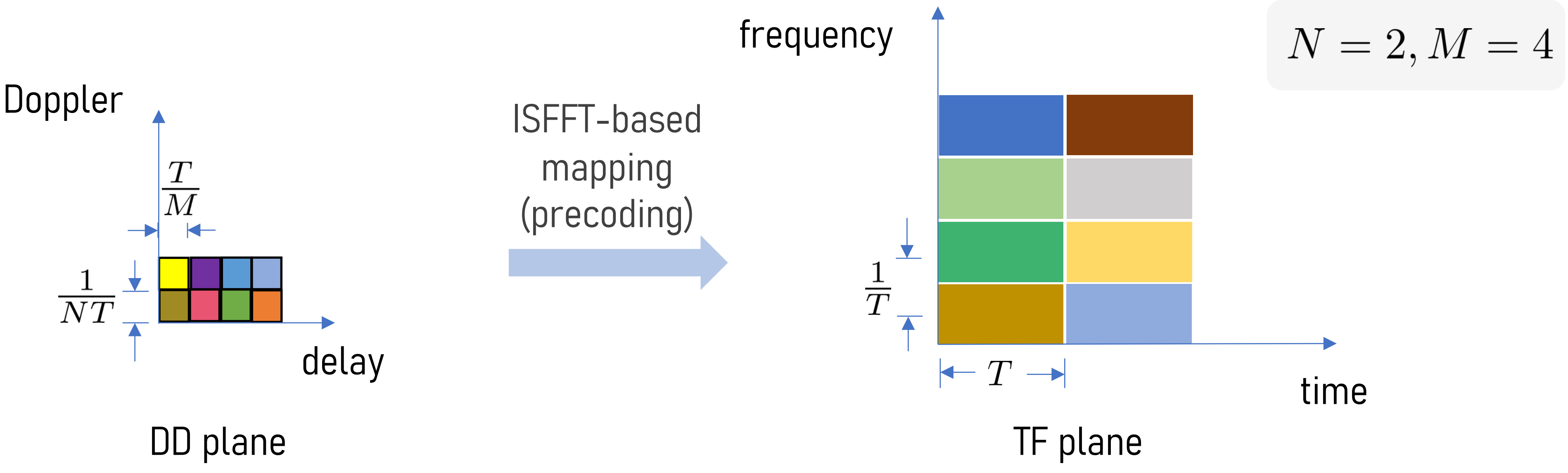
Introduction-OTFS Modulation



- Maps signals from DD plane to TF plane, then use OFDM
- OTFS waveform is orthogonal with respect to TF plane's resolution ($\mathcal{R} = \mathcal{TF} = 1$)
- OTFS's ideal pulse is assumed to satisfy **biorthogonal robust property**, which however cannot be realized.
- OTFS with rectangular pulse suffers high OOBE, complicated ISI and ICI

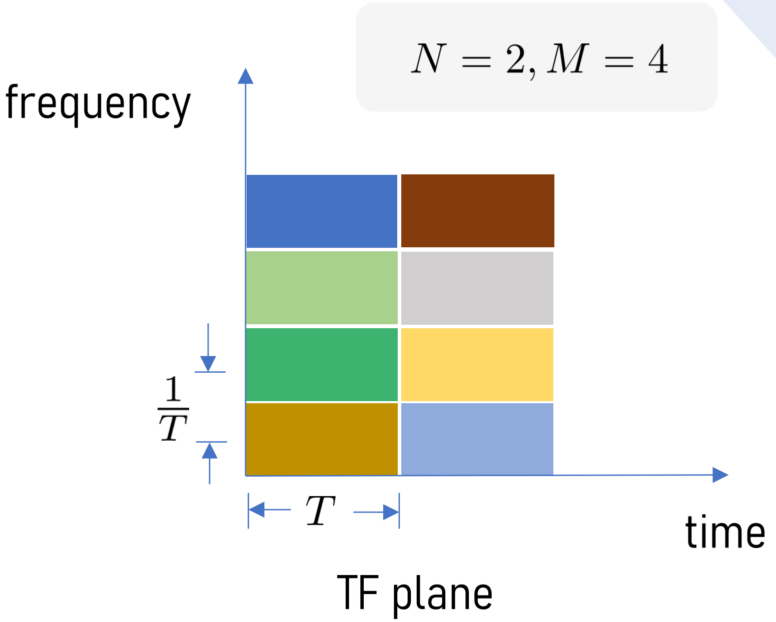
Introduction-Multicarrier (MC) Modulation

- MC Modulation

Transmit Pulses (Filters)

$$x(t) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} X_{m,n}\, g(t - m\mathcal{T})\, e^{j2\pi n \mathcal{F}(t - m\mathcal{T})}$$

Digital Symbol (QAM etc.) TF Lattice



- Pulse is a fundamental element of any modulation
  - defines the shape of signal in both time and frequency domains
  - determines how energy spread over the time and frequency
  - affects signal characteristics significantly (efficiency, localization, orthogonality, etc.)
- This talk
  - Introduce DDMC modulation
  - Design orthogonal pulses w.r.t DD resolutions
  - A promising waveform for ISAC

Propagation Channel with DD Domain Representation

LTV channels in the time-delay, TF, and DD domains

Time-variant impulse response

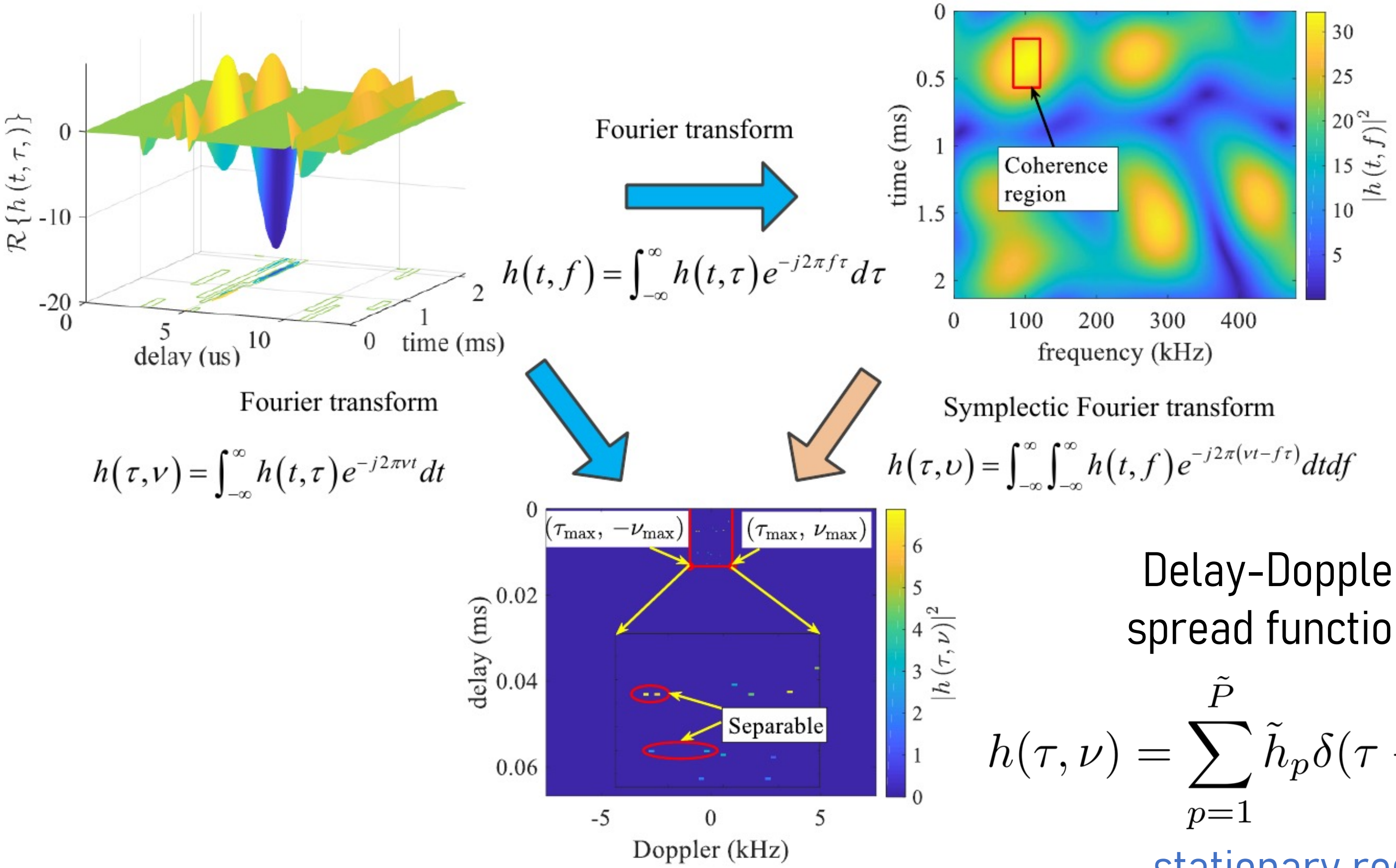
$h(t,\tau)$

Input delay spread function

Time-variant Transfer function

$h(t,f)$

coherence region
(2.5ms @ 5GHz)



Delay-Doppler spread function

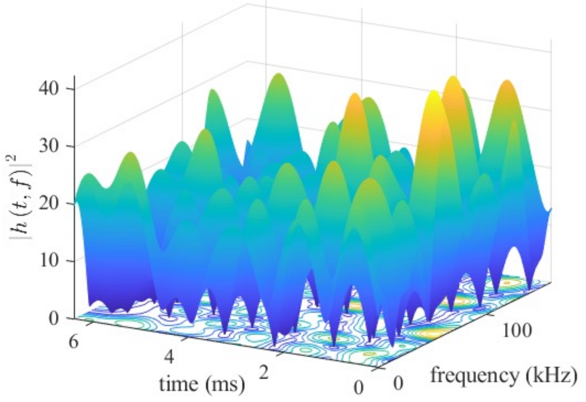
$$h(\tau,\nu)=\sum_{p=1}^{\tilde{P}}\tilde{h}_p\delta(\tau-\tilde{\tau}_p)\delta(\nu-\tilde{\nu}_p)$$

stationary region
(23 ms, 1479 ms @ 5GHz [Paier2008])

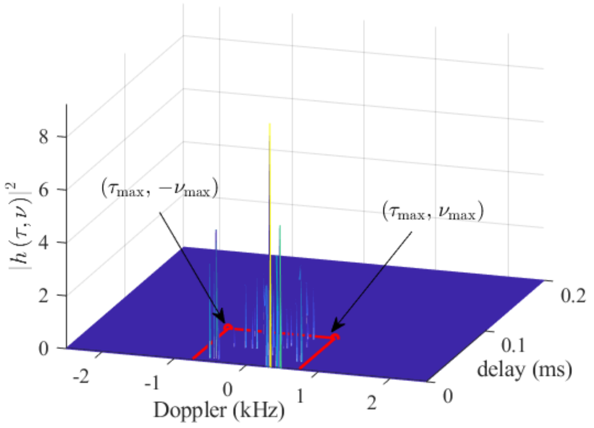
- Paradigm Shift
  - from TF to DD domain
  - from coherence region to stationary region

Propagation Channel with DD Domain Representation

Time-variant channel in high-mobility environments



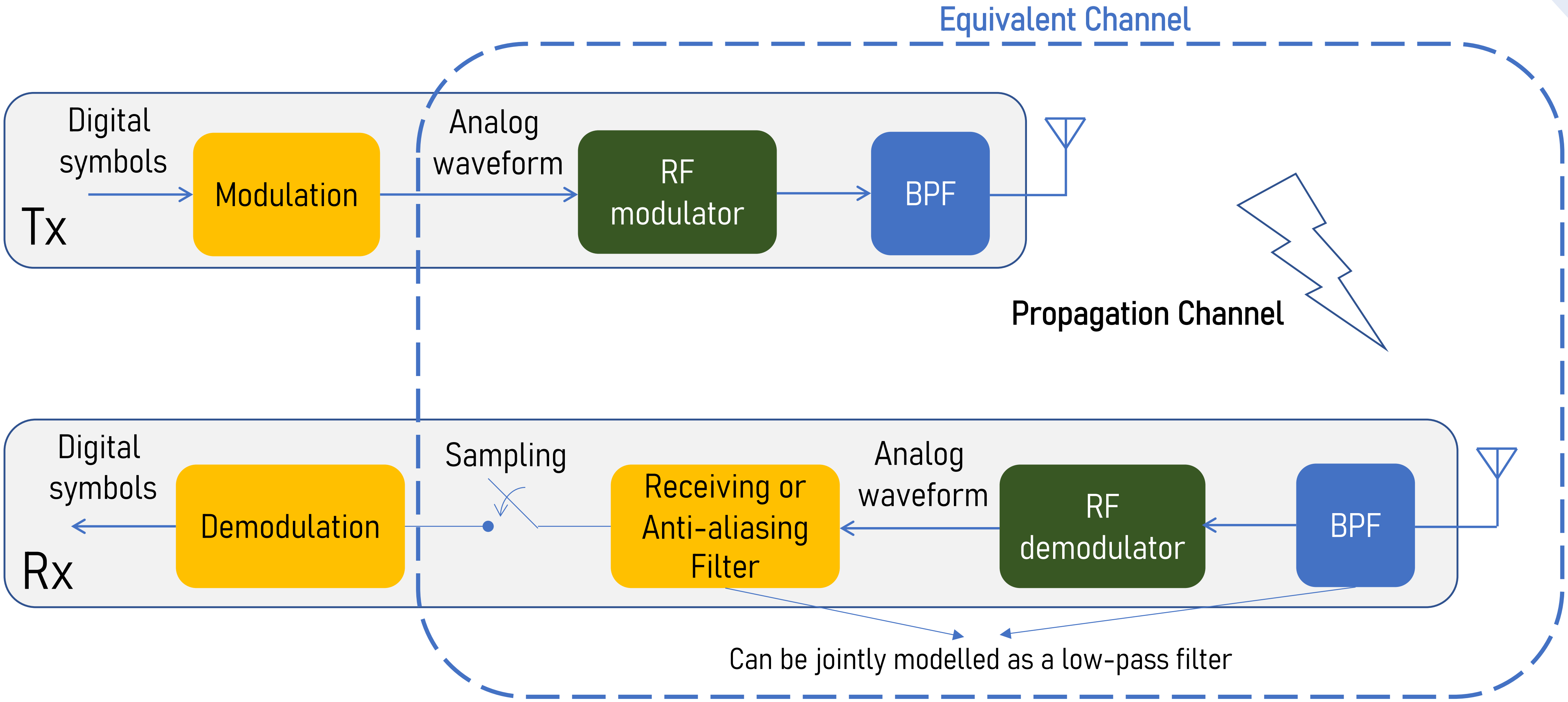
Time-frequency (TF) domain



Delay-Doppler (DD) domain

- TF domain channel is dense and changes quickly, difficult to perform estimation
- DD domain channel is compact and stable, allows accurate and low overhead channel estimation
- Deterministic model: delay-Doppler spread function, namely spreading function
- Path based model : $\tilde{h}(\tau,\nu) = \sum_{p=1}^{\tilde{P}} \tilde{h}_p \delta(\tau - \tilde{\tau}_p)\delta(\nu - \tilde{\nu}_p)$

Sampled Channel Model with Combined TF Constraints



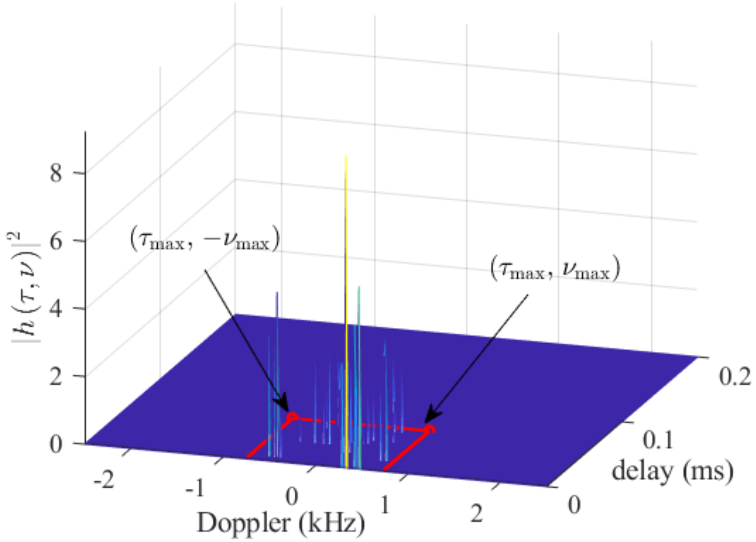
• P. Bello, "Characterization of randomly time-variant linear channels," IEEE Trans. Commun. Syst., vol. 11, no. 4, pp. 360–393, 1963.

Sampled Channel Model with Combined TF Constraints

- "All real-life channels and signals have an essentially finite number of DoF, due to restrictions on time duration and bandwidth." [Bello, 1963].
- "All radio communications systems have a finite delay resolution related to the reciprocal of their transmission bandwidths." "finite Doppler resolution" [Steele & Hanzo, Mobile Radio Communications,1999]
- Sampled channel model with combined time and frequency constraints (discrete equivalent on-the-grid channel model)

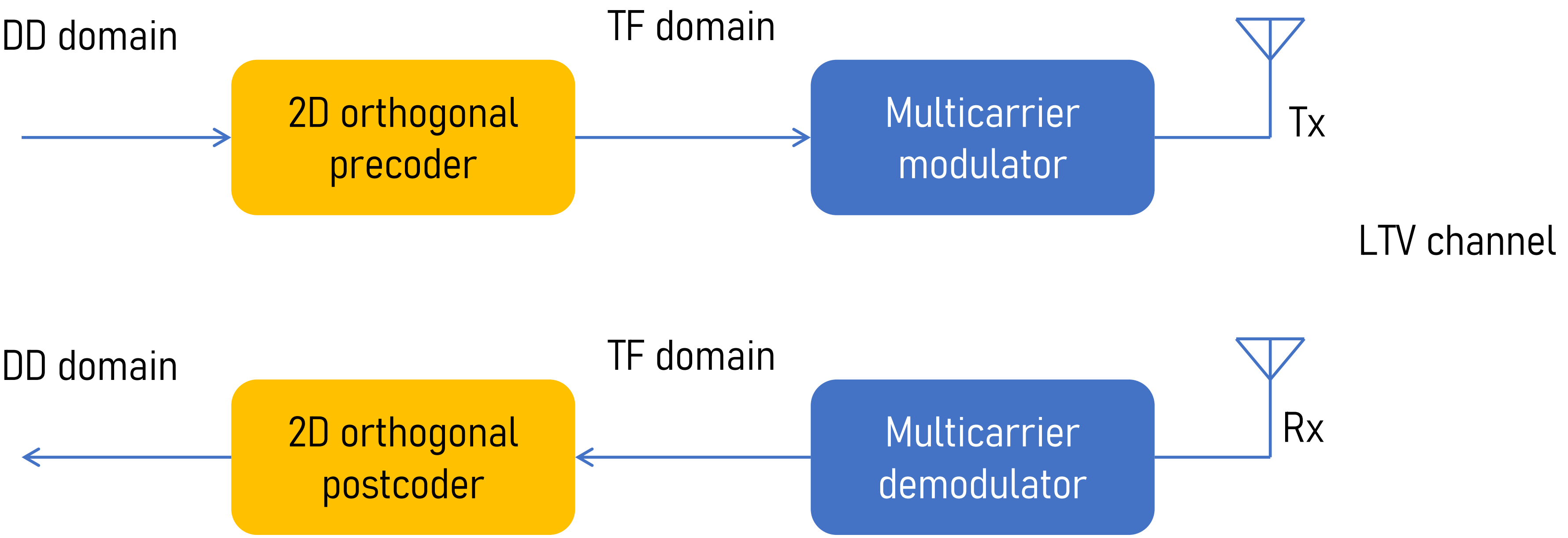
$$h(\tau,\nu)=\sum_{p=1}^{P}h_p\delta(\tau-\tau_p)\delta(\nu-\nu_p)$$

$$\tau_p=\frac{l_p}{M\Delta f},\nu_p=\frac{k_p}{NT}$$



- $M\Delta F$ is the sampling rate ➔ delay resolution being $1/(M\Delta F)$
- NT is the frame duration ➔ Doppler resolution being $1/NT$
- l_p: delay index; k_p : Doppler index (Neither fractional delay nor fractional Doppler)

OTFS Modulation



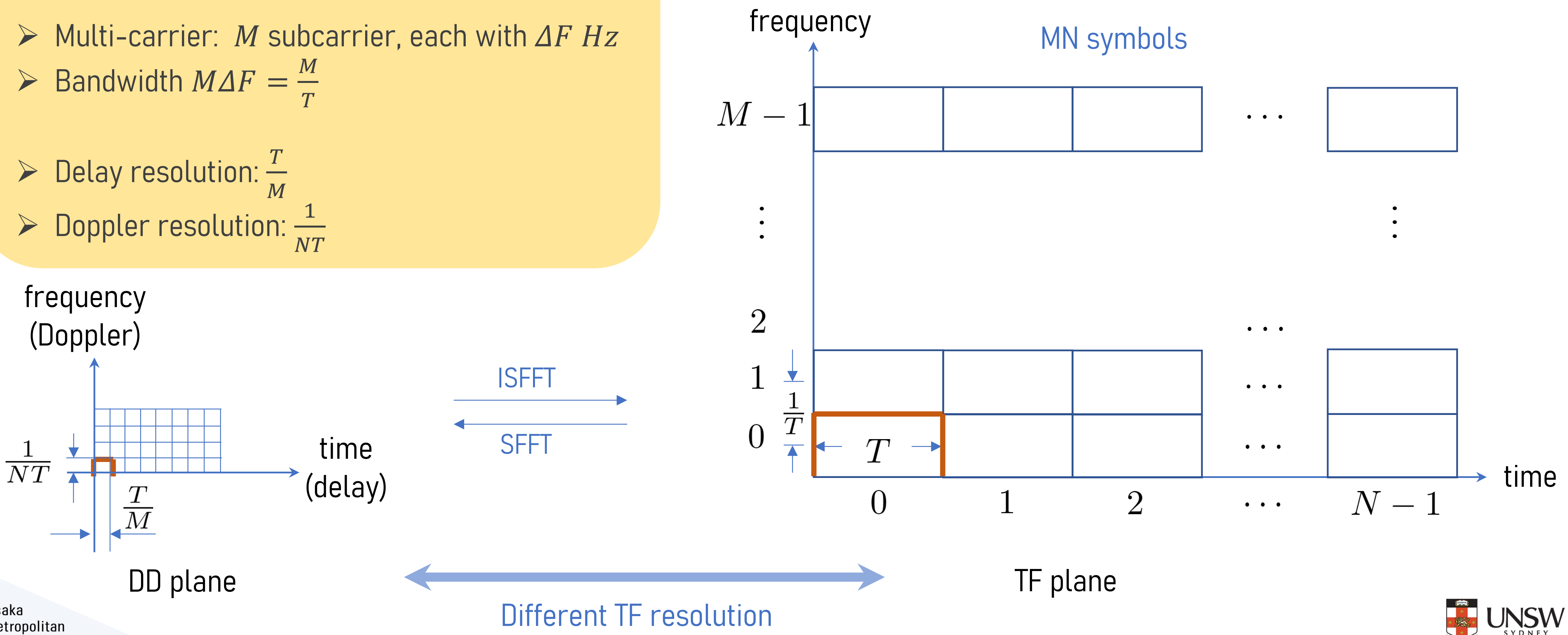
- ✓ 2D orthogonal precoding from DD domain to TF domain, such as inverse Symplectic Finite Fourier Transform (ISFFT) and Walsh-Hadamard transform
- ✓ Multi-carrier modulator from TF domain to time domain, such as OFDM and FBMC
- ✓ OTFS relies on its employed TF domain MC modulation waveform

OTFS Modulation

- A frame: N time slots, each with $T\ sec$
- Frame length NT

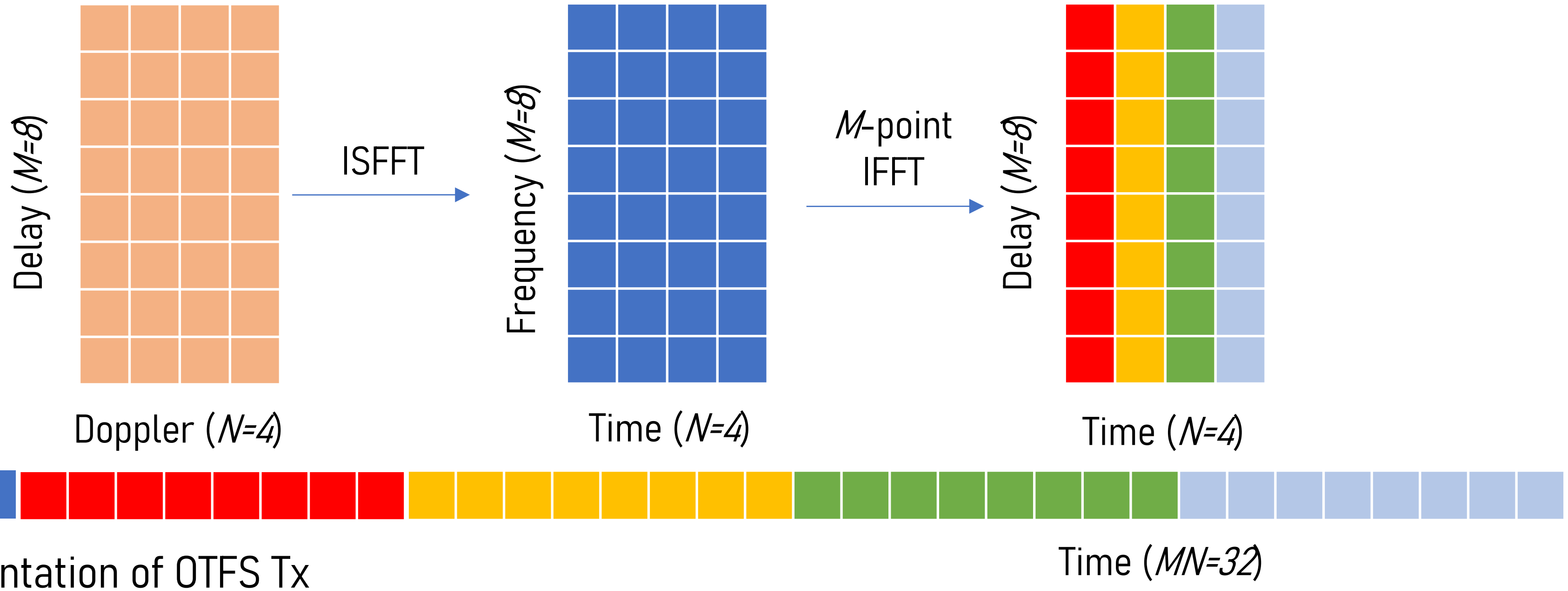
- Multi-carrier: M subcarrier, each with $\Delta F\ Hz$
- Bandwidth $M\Delta F = \frac{M}{T}$

- Delay resolution: $\frac{T}{M}$
- Doppler resolution: $\frac{1}{NT}$

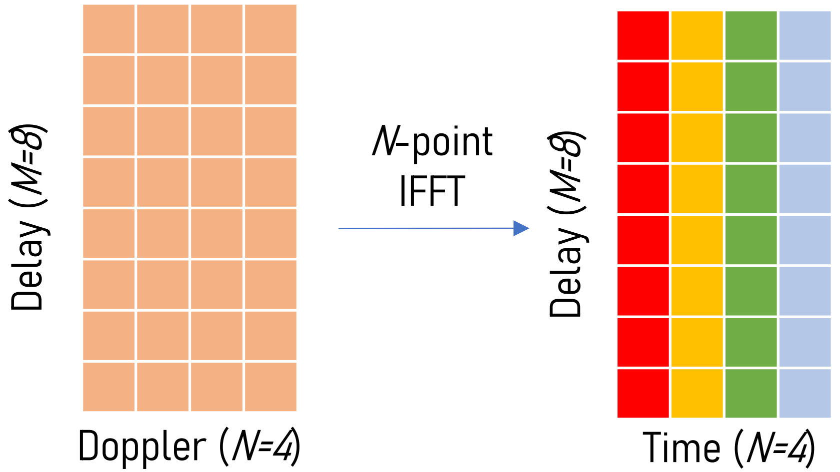


OTFS Modulation

- Direct implementation of OTFS Tx



- Simple implementation of OTFS Tx



- R. Hadani, "OTFS: A novel modulation scheme addressing the challenges of 5G," YouTube video, October 22, 2018.

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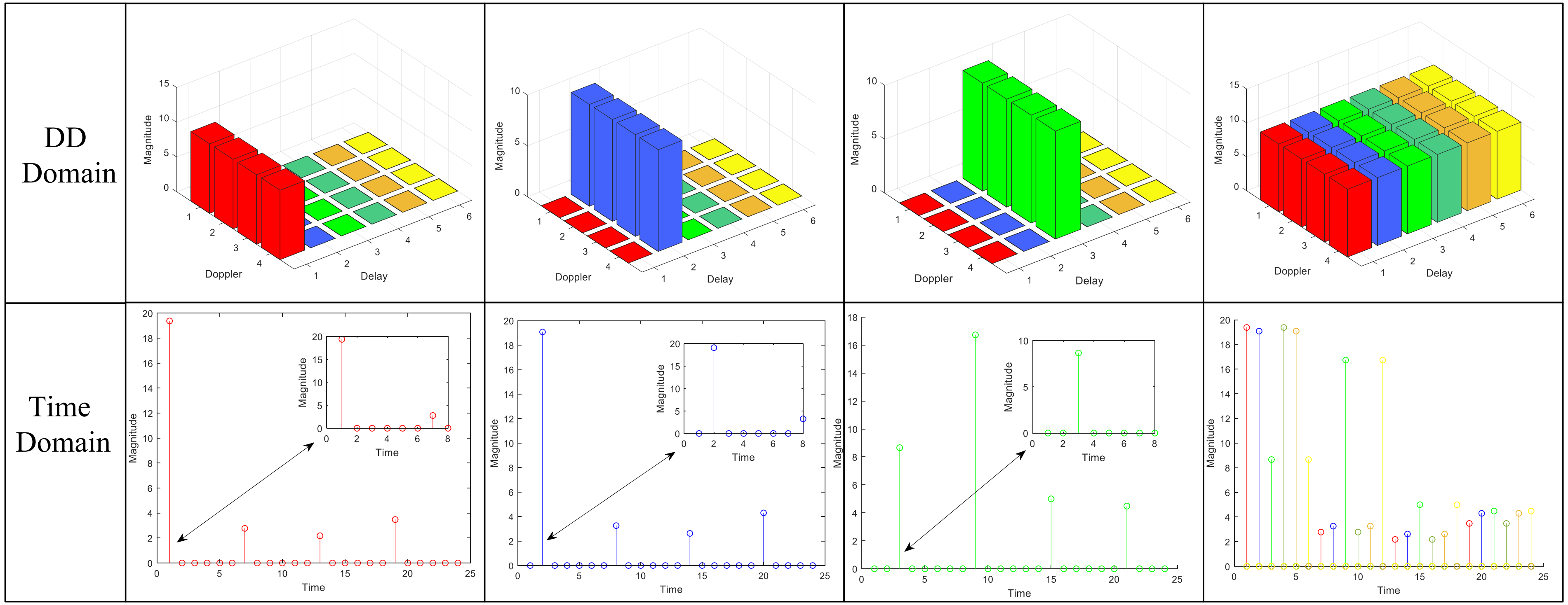
OTFS Modulation

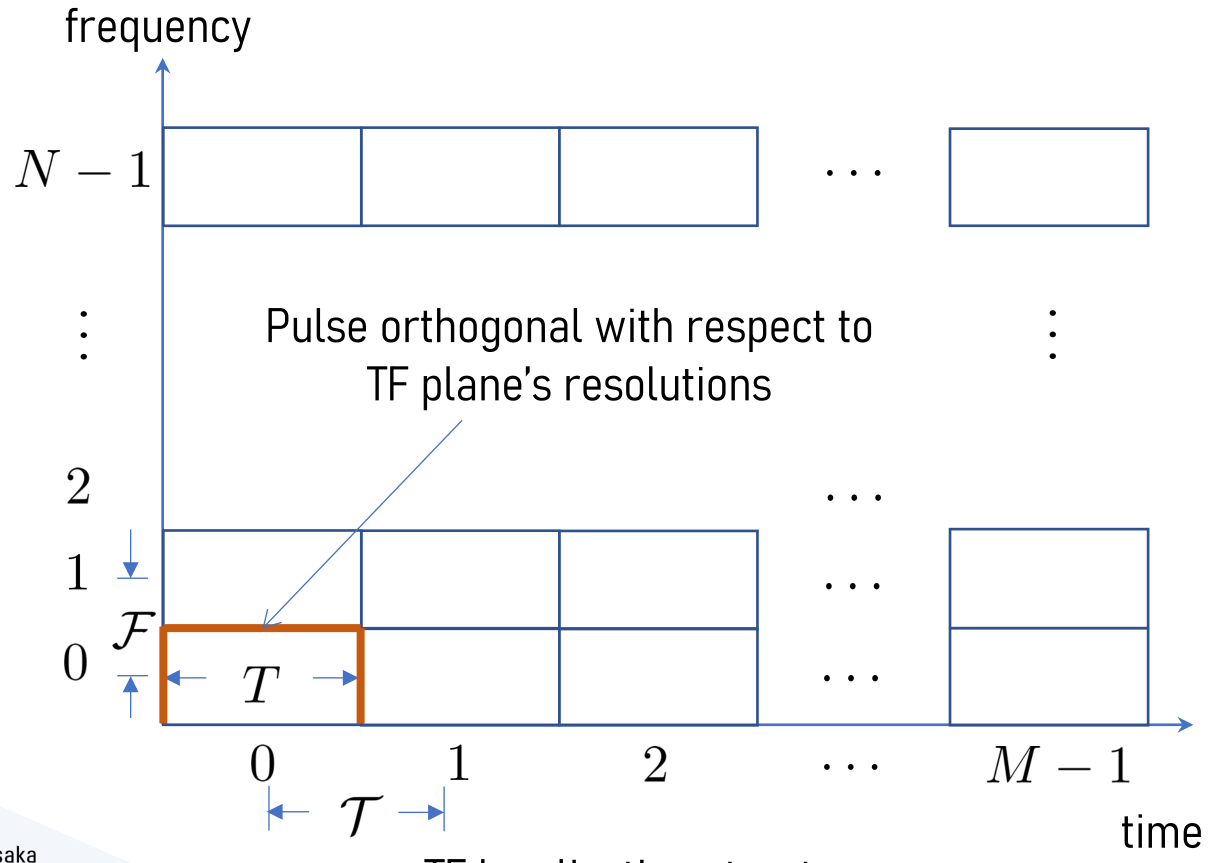


- Connection between OTFS and OFDM
  - OTFS can be overlayed on OFDM, therefore no new pulse design.
  - IDFT from Doppler domain to Time domain suggests a potential MC.
  - Looks like multiple MC signals staggered with delay division.

Is it possible to design a DD plane orthogonal pulse ?

Waveform/Pulse Design Principles for MC modulation

- A modulation waveform is a continuous-time function (analog signal)
- One analog pulse carriers one digital symbol
- MN symbols ⇒ MN pulses orthogonal with each other to synthesis the whole waveform



TF localization structure

time resolution (symbol interval) — frequency resolution (subcarrier spacing)

$$g_{m,n}(t) = g(t - m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$
$$0 \le m \le M-1, 0 \le n \le N-1$$

Joint TF resolution (JTFR) : $\mathcal{R} = \mathcal{T}\mathcal{F}$

Symbol period : $T = \frac{1}{\mathcal{F}}$

prototype pulse/filter

- Core of MC modulation: $g(t)$ given $(\mathcal{T}, \mathcal{F})$

Waveform/Pulse Design Principles for MC modulation

- An MC modulation is defined by prototype pulse $g(t)$ and the TF lattice $(\mathcal{T}, \mathcal{F})$
- Traditionally, $g_{m,n}(t)$'s are treated as Weyl-Heisenberg (WH) or Gabor function set
- Design an MC modulation : ⇒ Find (bi)orthogonal WH/Gabor sets ⇒ Find $\boldsymbol{g(t)}$ given $(\mathcal{T}, \mathcal{F})$
- OFDM: Rectangular pulse for $\mathcal{R} = \mathcal{T}\mathcal{F} = 1$, symbol interval = symbol period
- Fundamental of MC modulation can be found in the following two tutorial papers

- G. Matz, H. Bolcskei, and F. Hlawatsch, "Time-frequency foundations of communications: Concepts and tools," IEEE Signal Process. Mag., 2013.
- A. Sahin, I. Guvenc, and H. Arslan, "A survey on multicarrier communications: Prototype filters, lattice structures, and implementation aspects," IEEE Commun. Surveys Tuts., 2014.

(Bi)Orthogonal WH/Gabor Sets

- Fundamental tool of time-frequency analysis （TFA） for signals/functions
- Gabor (Weyl-Heisenberg, Short-time/Windowed Fourier) expansion
- For signals lie in space $L^2(\mathbb{R})$

$$x(t) = \sum_{m\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} c_{m,n} g_{m,n}(t), \quad g_{m,n}(t) = g(t-m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$

$g(t)$: Gabor atom (function), prototype pulse

$$\hat{c}_{m,n} = \langle x, \gamma_{m,n}\rangle = \int x(t)\gamma^*_{m,n}(t)dt$$

$$\gamma_{m,n}(t) = \gamma(t-m\mathcal{T})e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$

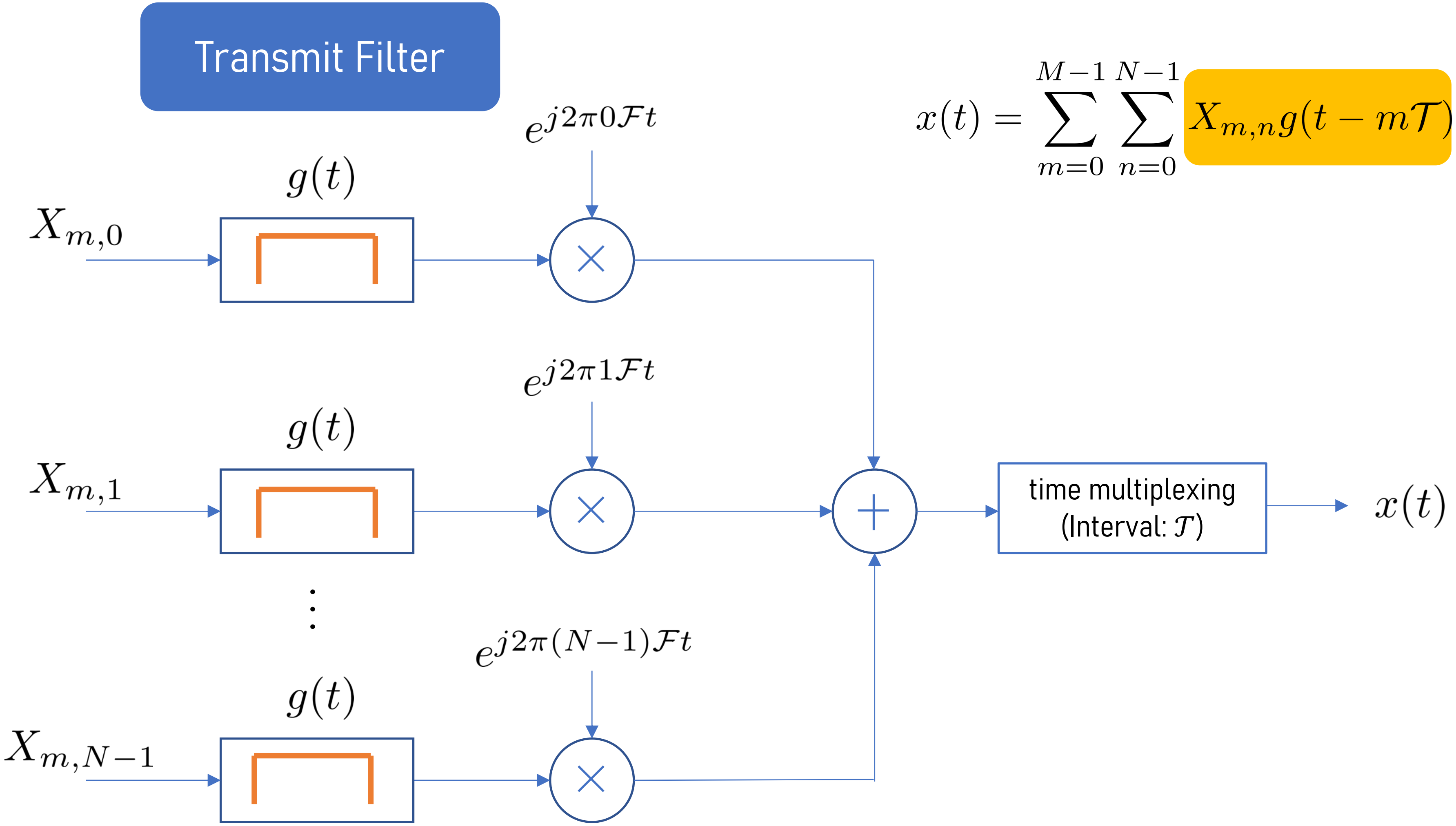
- WH sets: $(g,\mathcal{T},\mathcal{F}) = \{g_{m,n}(t)\}_{m,n\in\mathbb{Z}}$, $(\gamma,\mathcal{T},\mathcal{F}) = \{\gamma_{m,n}(t)\}_{m,n\in\mathbb{Z}}$
- WH frames: Complete or overcomplete WH sets with guaranteed numerical stability of reconstruction

JTFR	Sampling	Completeness	Frame for $(g,\frac{1}{\mathcal{F}},\frac{1}{\mathcal{T}}), (\gamma,\frac{1}{\mathcal{F}},\frac{1}{\mathcal{T}})$	(Bi)orthogonal WH sets exist?
$\mathcal{R}=\mathcal{TF}>1$	Undercritical	Incomplete	✓ dual/tight	Yes
$\mathcal{R}=\mathcal{TF}=1$	Critical	Complete	✓ dual/tight	Yes
$\mathcal{R}=\mathcal{TF}<1$	Overcritical	Overcomplete	× dual/tight	No

Direct Implementation of MC Modulation

- Requires N modulators therefore high-complexity

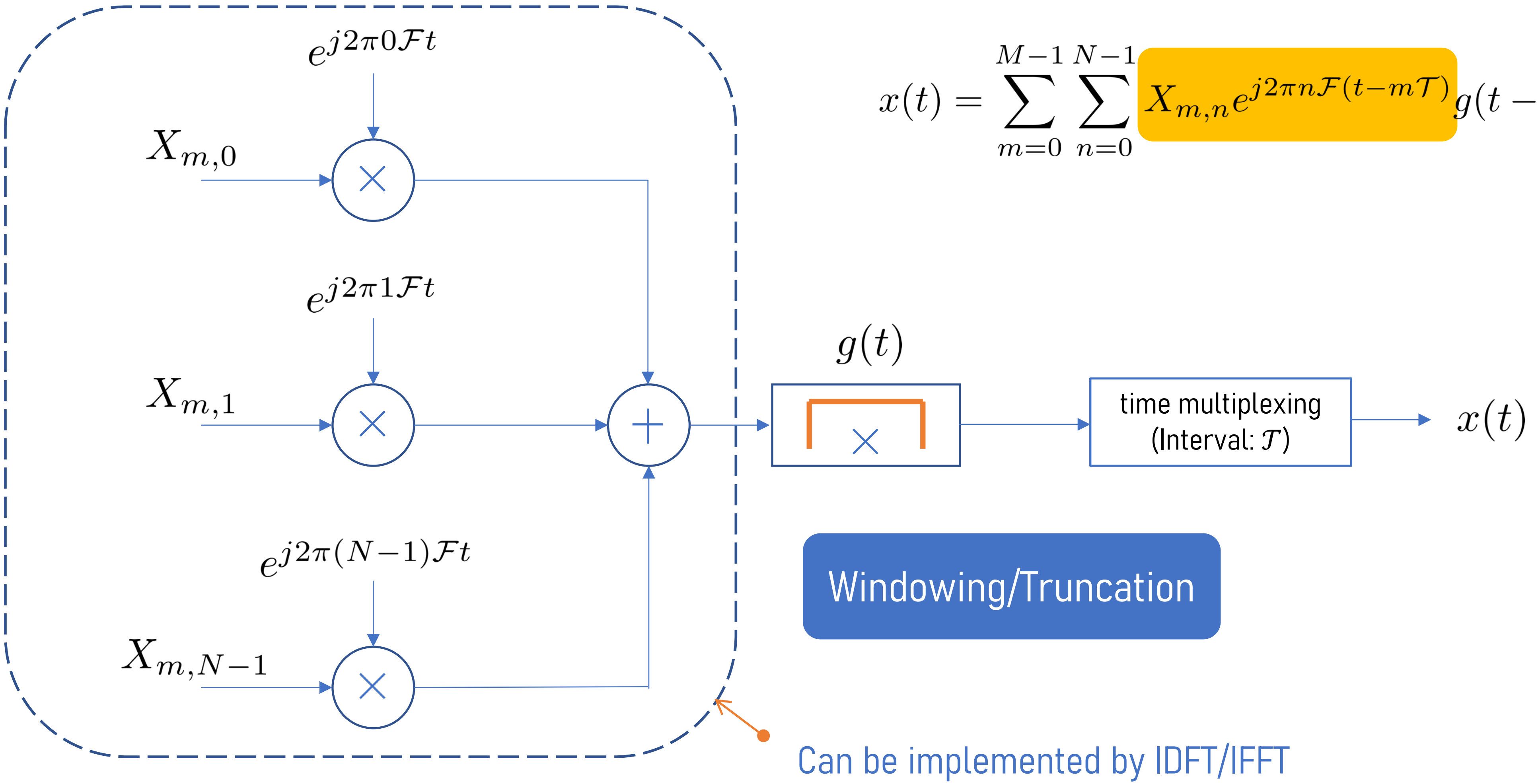
$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X_{m,n} g(t - m\mathcal{T}) e^{j2\pi n\mathcal{F}(t-m\mathcal{T})}$$



Direct Implementation of MC Modulation

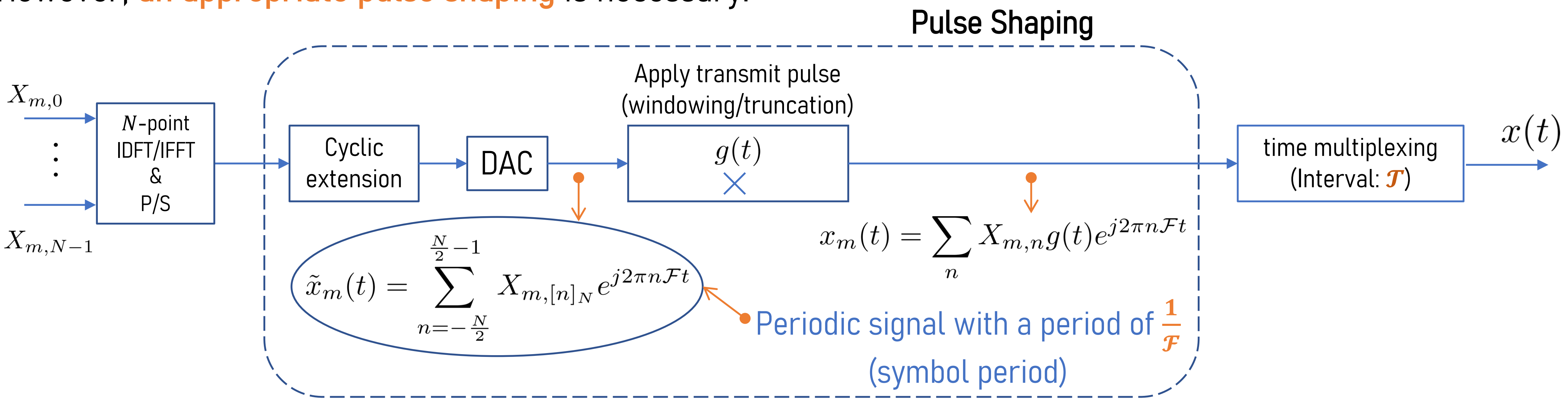
- Requires N modulators therefore high-complexity

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X_{m,n} e^{j2\pi n \mathcal{F}(t-m\mathcal{T})} g(t - m\mathcal{T})$$



IDFT-based Implementation of MC Modulation

- Use IDFT to achieve a low-complexity implementation [Weinstein, 1971]
- However, an appropriate pulse shaping is necessary.

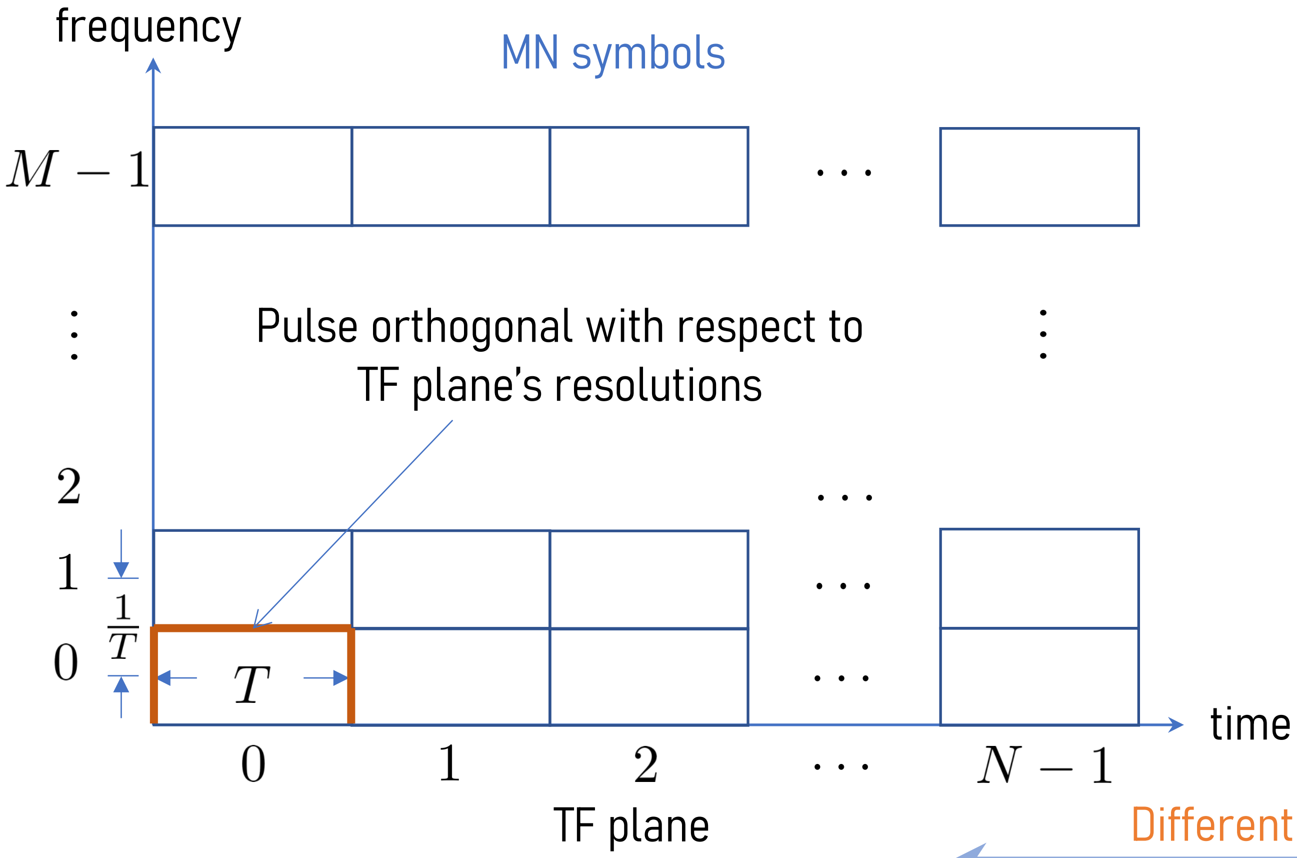


- IDFT/IFFT is just a step of one of the implementation methods of MC modulation
- A detailed explanation of OFDM (MC modulation) pulse shaping can be found at Section II.C of
  - H. Lin and J. Yuan, “Multicarrier Modulation on Delay-Doppler Plane: Achieving Orthogonality with Fine Resolutions,” IEEE ICC 2022.

Fundamental Issue of DDMC Modulation

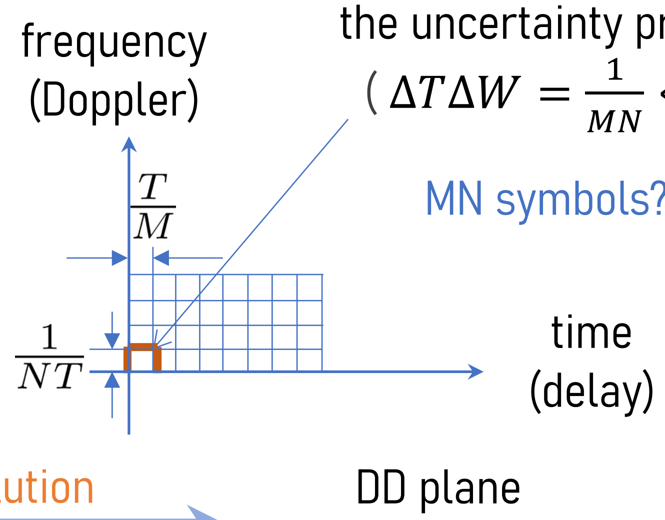
- Principle : one pulse for one symbol

DD plane orthogonal pulse ???



This "tiny" pulse violates the uncertainty principle. ($\Delta T \Delta W = \frac{1}{MN} \ll \frac{1}{4\pi}$)

MN symbols?



Different TF resolution

The DD plane is also a TF plane however with high resolutions.

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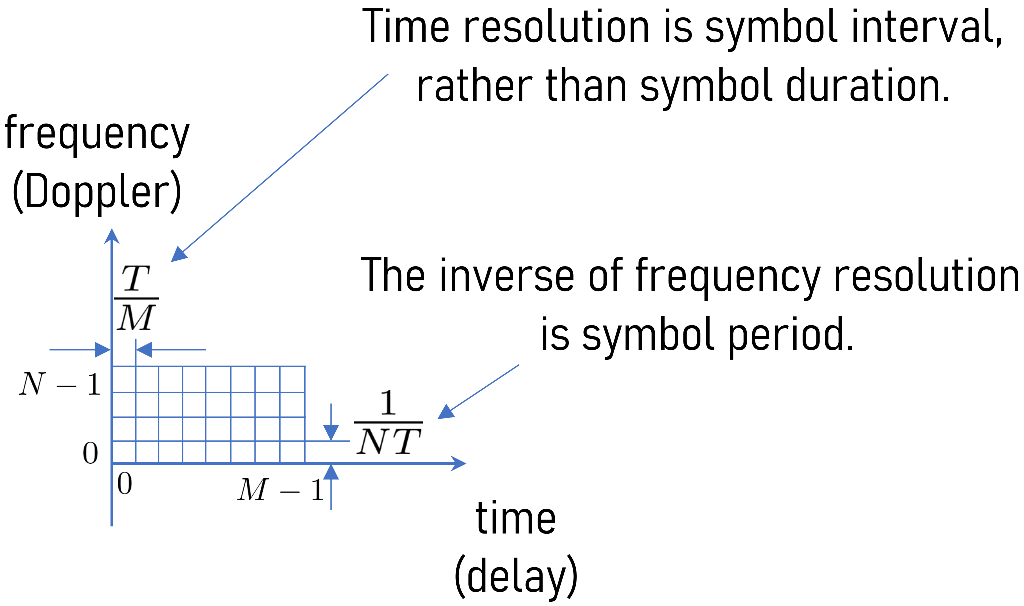
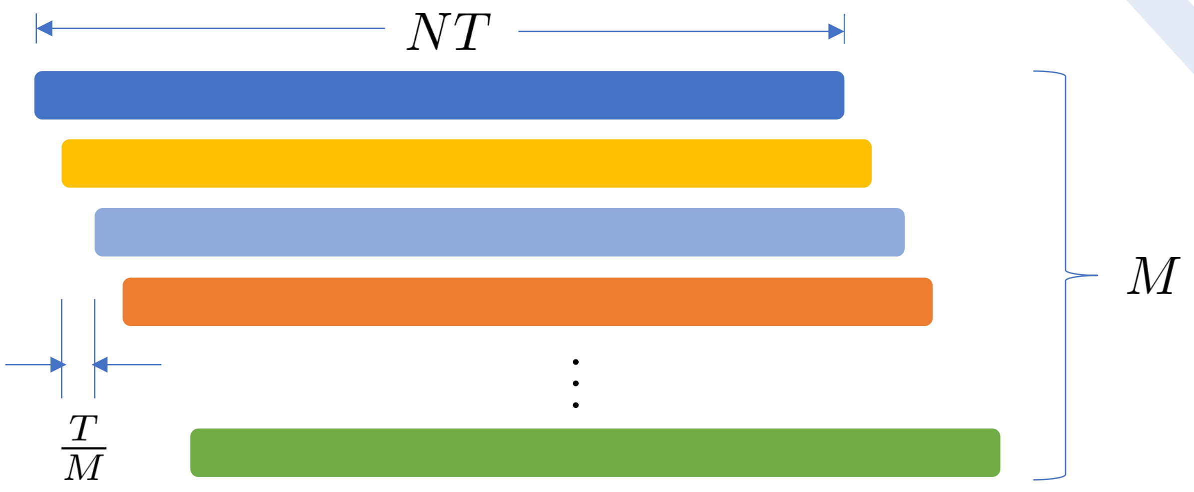
DDMC Modulation



TF lattice in DD plane

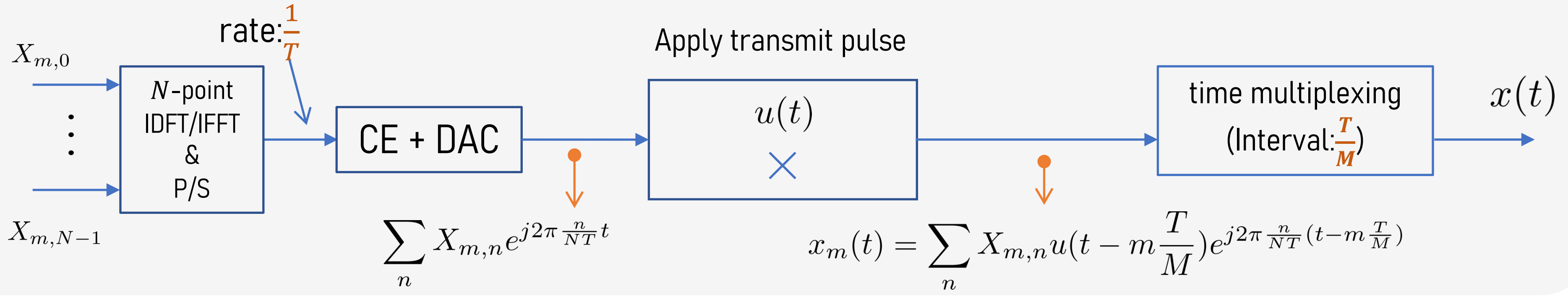


M MC symbols (NT-length, N subcarriers) stagger/overlap.

- DDMC modulation ⇒ A type of staggered multitone (SMT) modulation
- Short symbol interval ⇒ Wideband signal.
- Long symbol period ⇒ Narrowband MC signal, How is this possible?

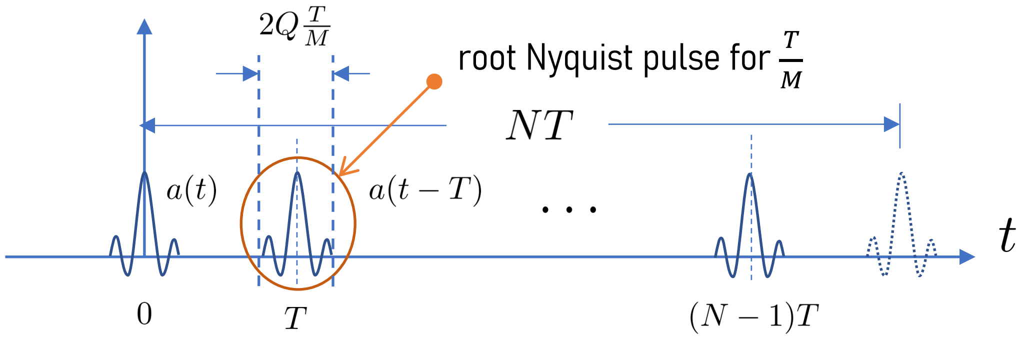
Transmit Pulse for DDMC Modulation

- Symbol interval $\frac{T}{M}$ $\ll$ Symbol period NT



- DD Plane Orthogonal Pulse (DDOP)

$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T)$$



Ambiguity function of $u(t)$

$$A_{u,u}\left(m\frac{T}{M}, n\frac{1}{NT}\right) = \delta(m)\delta(n), \forall |m| \le M-1, |n| \le N-1$$

Local (Bi)Orthogonality for WH Subsets

- WH sets: $(g, \mathcal{T}, \mathcal{F}) = \{g_{m,n}(t)\}_{m,n\in\mathbb{Z}}, \ (\gamma, \mathcal{T}, \mathcal{F}) = \{\gamma_{m,n}(t)\}_{m,n\in\mathbb{Z}}$

- WH subset: $$(g, \mathcal{T}, \mathcal{F}, M, N) = \{g_{m,n}(t)\}_{0\le m\le M-1, 0\le n\le N-1}$$ $$(\gamma, \mathcal{T}, \mathcal{F}, M, N) = \{\gamma_{m,n}(t)\}_{0\le m\le M-1, 0\le n\le N-1}$$

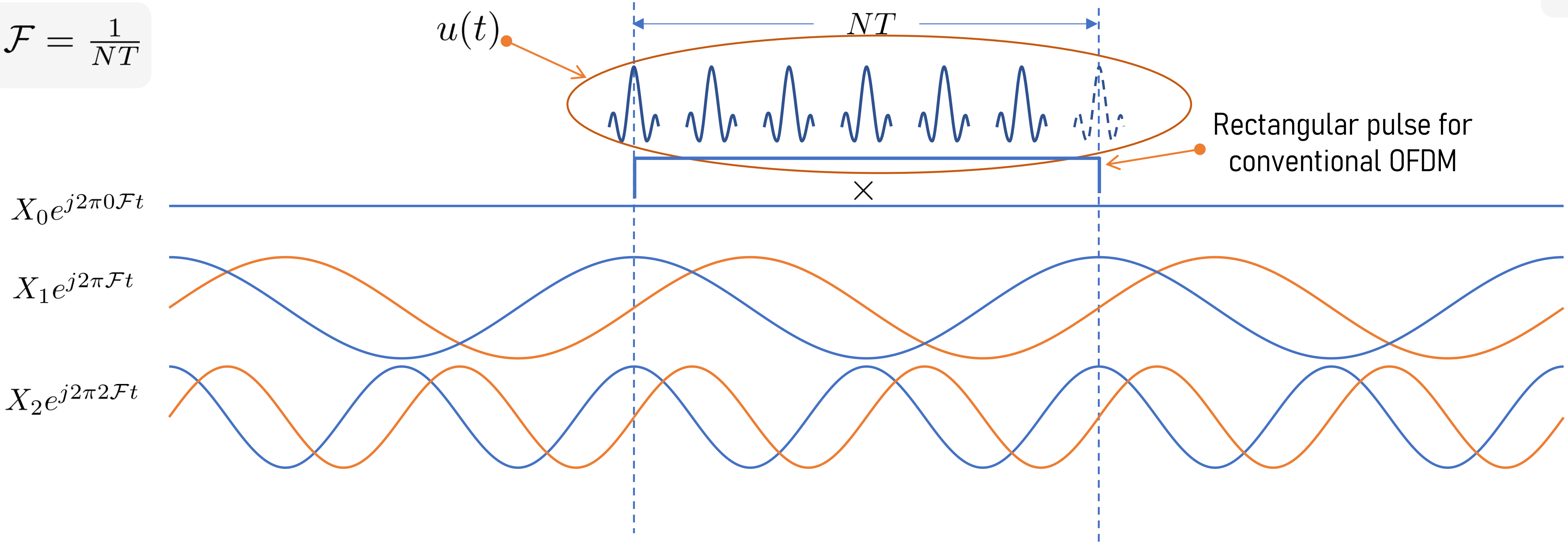
- (Bi)Orthogonality among WH sets:
  - $m, n \in \mathbb{Z} \Rightarrow$ Global (bi)orthogonality governed by the WH frame theory

- (Bi)Orthogonality among WH subsets:
  - $0 \le m \le M-1, 0 \le n \le N-1 \ \Rightarrow$ Local (bi)orthogonality
  - Local (bi)orthogonality is **not** necessarily governed by the WH frame theory
  - Local (bi)orthogonality is **enough for a modulation in the TF region of interest**.

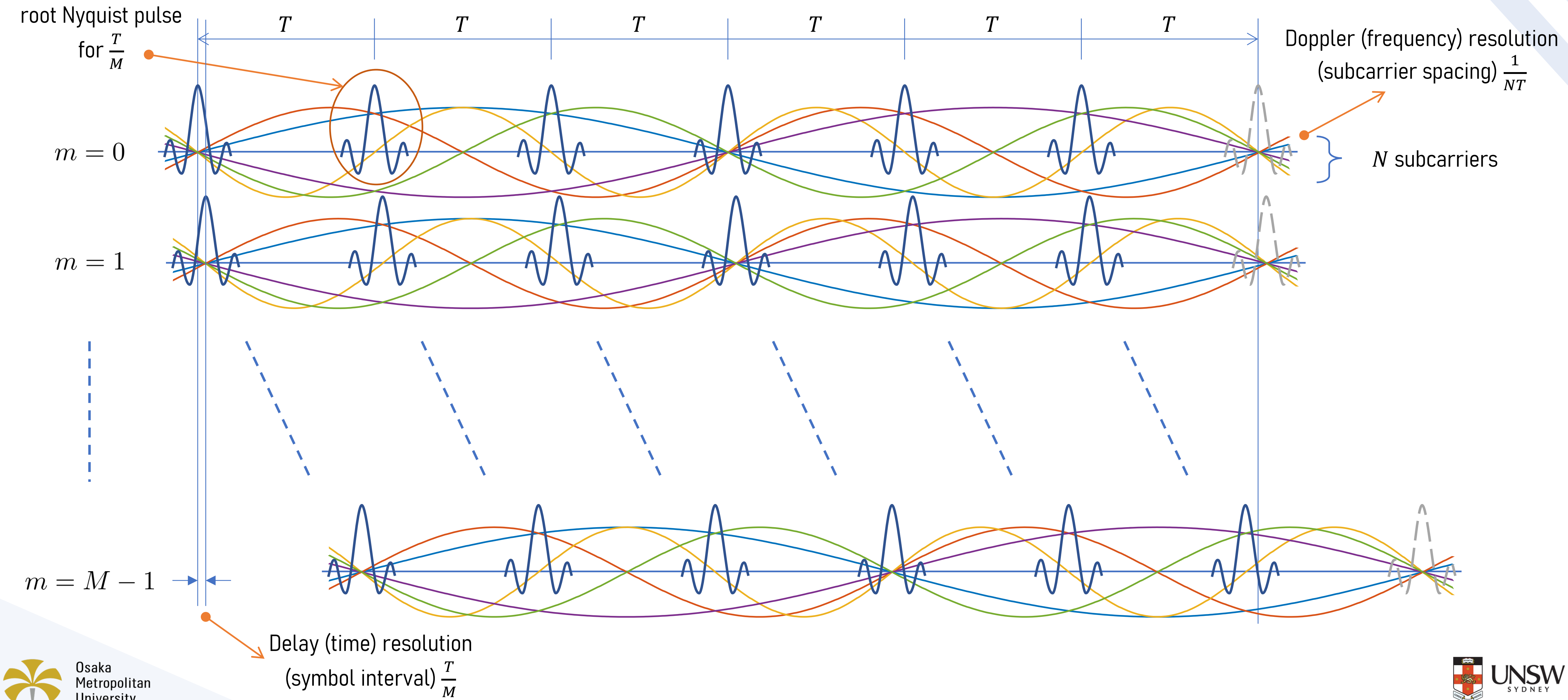
Orthogonal Delay-Doppler Division Multiplexing (ODDM)

$$\mathcal{F} = \frac{1}{NT}$$

$N = 6$



Orthogonal Delay-Doppler Division Multiplexing (ODDM)



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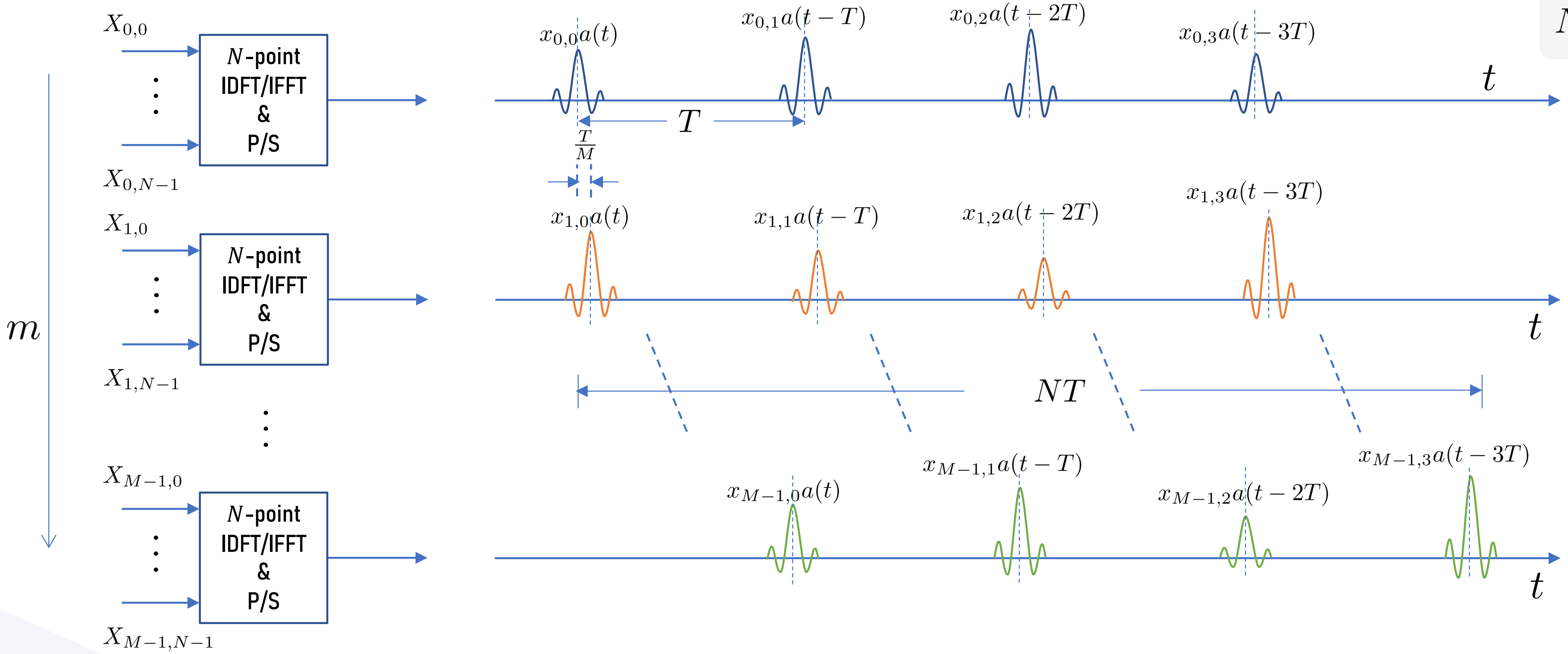
Low-Complexity Implementation of ODDM Modulation

- ODDM is pulse-shaped by $u(t)$ and therefore orthogonal with respect to the DD plane's resolutions $\frac{T}{M}$ and $\frac{1}{NT}$.
- When $2Q \ll M$, the combination of N-point IDFT/IFFT and $a(t)$-based filtering **approximates** ODDM waveform.

$N = 4, 2Q \ll M$



- Add a frame-wise CP to keep the periodicity

ODDM Waveform versus OTFS Waveform

ODDM:

$$x(t)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}X[m,n]u\left(t-m\frac{T}{M}\right)e^{j2\pi\frac{n}{NT}\left(t-m\frac{T}{M}\right)}$$

$$u(t)=\sum_{\dot{n}=0}^{N-1}a(t-\dot{n}T)$$

Transmit pulse of ODDM

$$A_{u,u}\left(m\frac{T}{M},n\frac{1}{NT}\right)=\delta(m)\delta(n)$$

$$\forall|m|\leq M-1,|n|\leq N-1$$

OTFS:

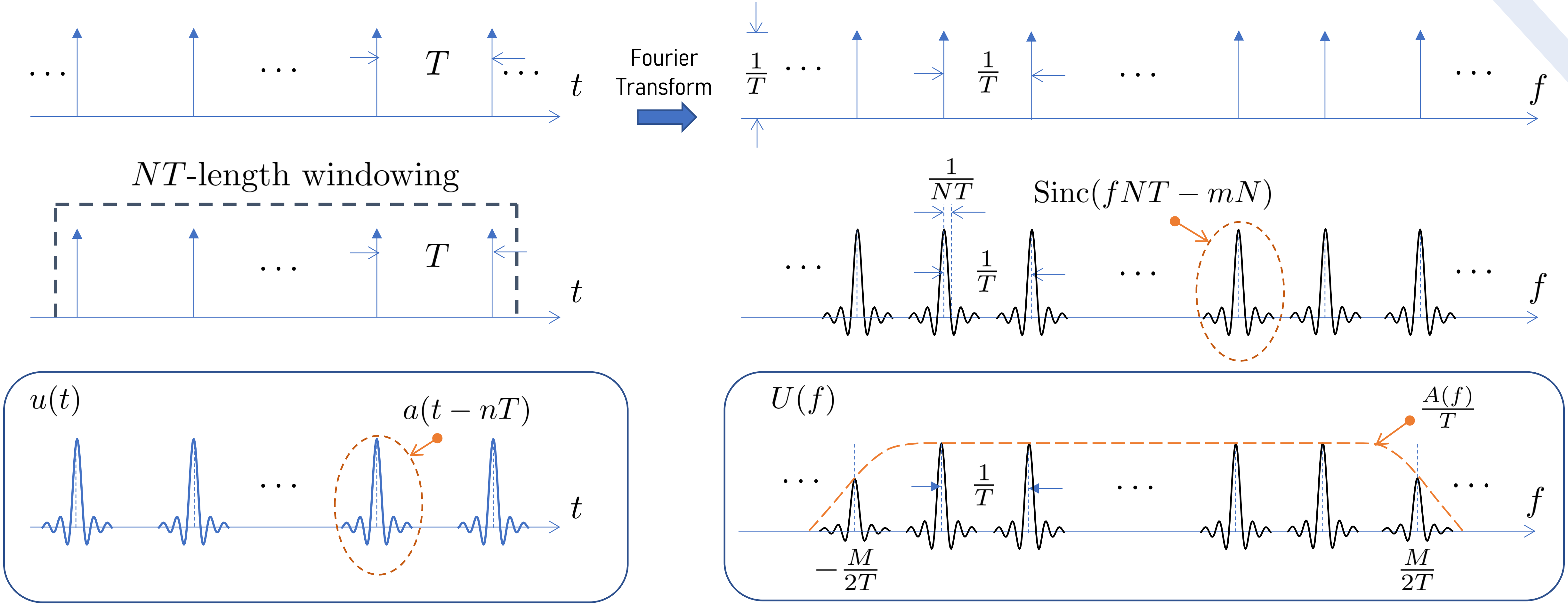
Transmit pulse of OFDM

$$s(t)=\sum_{\grave{n}=0}^{N-1}\sum_{\grave{m}=0}^{M-1}\mathcal{X}[\grave{m},\grave{n}]g(t-\grave{n}T)e^{j2\pi\grave{m}\frac{1}{T}(t-\grave{n}T)}$$

$$\mathcal{X}[\grave{m},\grave{n}]=\frac{1}{\sqrt{MN}}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}X[m,n]e^{j2\pi\left(\frac{\grave{n}n}{N}-\frac{\grave{m}m}{M}\right)}$$

$$A_{g,g}\left(nT,m\frac{1}{T}\right)=\delta(m)\delta(n)$$

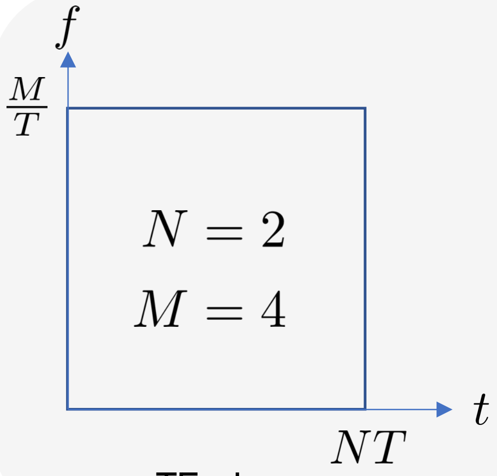
Frequency Domain Representation of DDOP



$$U(f) = \frac{e^{-j2\pi f\tilde{T}}}{T} A(f) \sum_{m=-\infty}^{\infty} e^{j2\pi \frac{m(N-1)}{2}} \mathrm{Sinc}(fNT - mN)$$

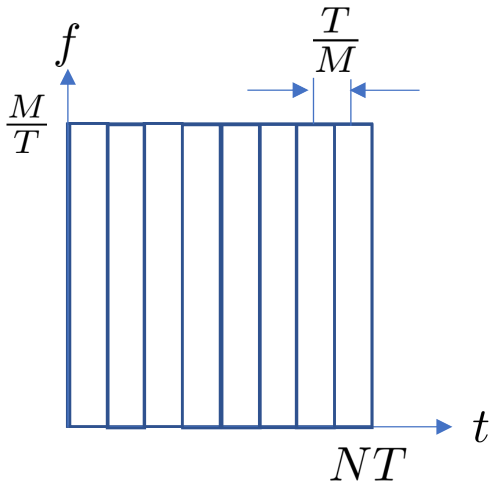
- Phase terms are ignored for the purpose of display

From the Viewpoint of DoF

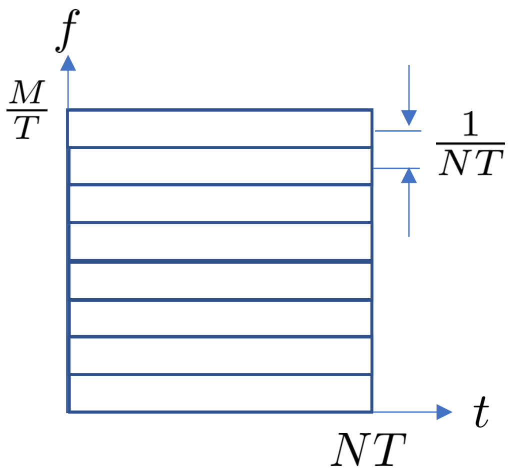


TF plane

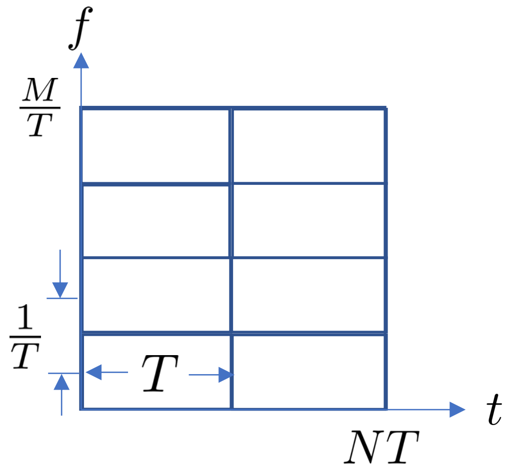
- Landau-Pollak Theorem
- DoF$\approx \frac{M}{T} \times NT = MN$
- MN symbols



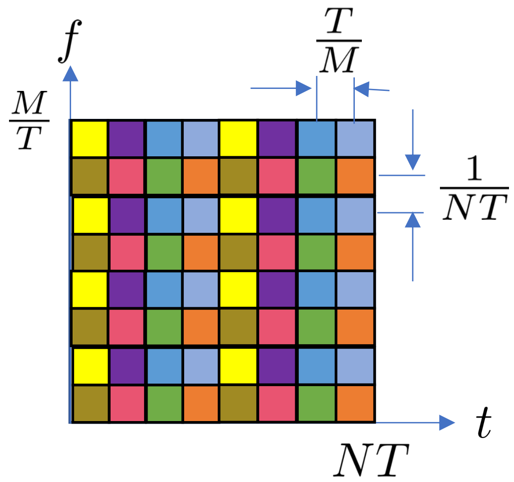
TDM(SC)
DoF: MN



FDM(OFDM)
DoF: MN



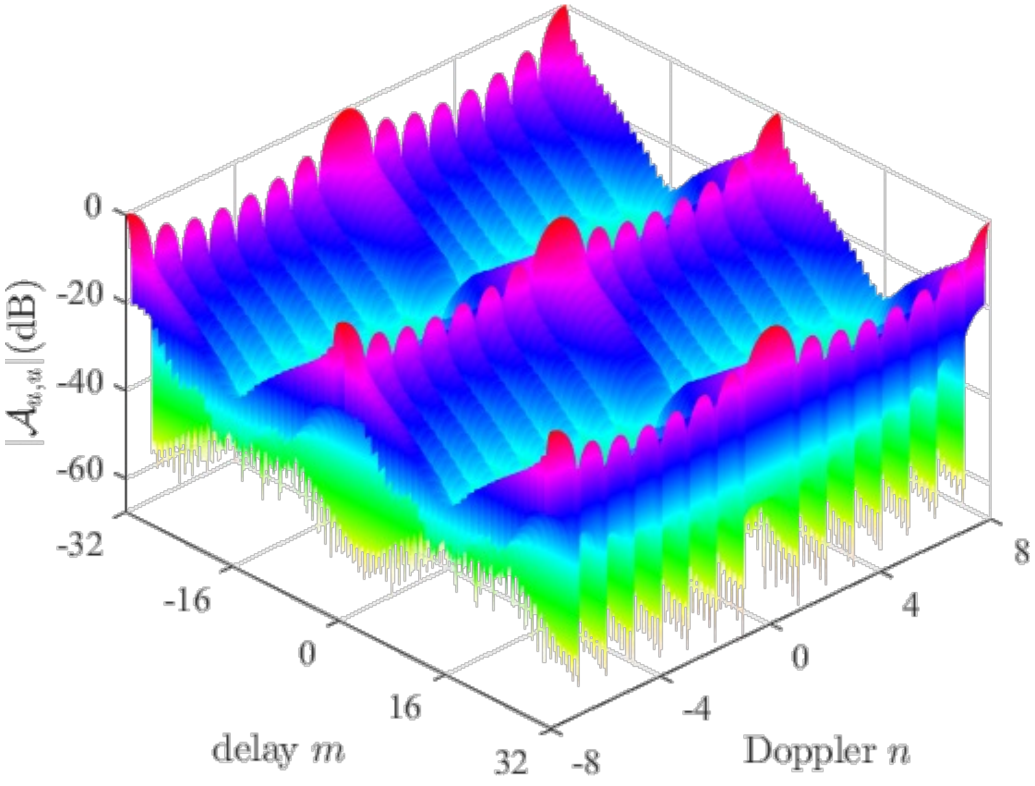
OFDM
DoF: MN



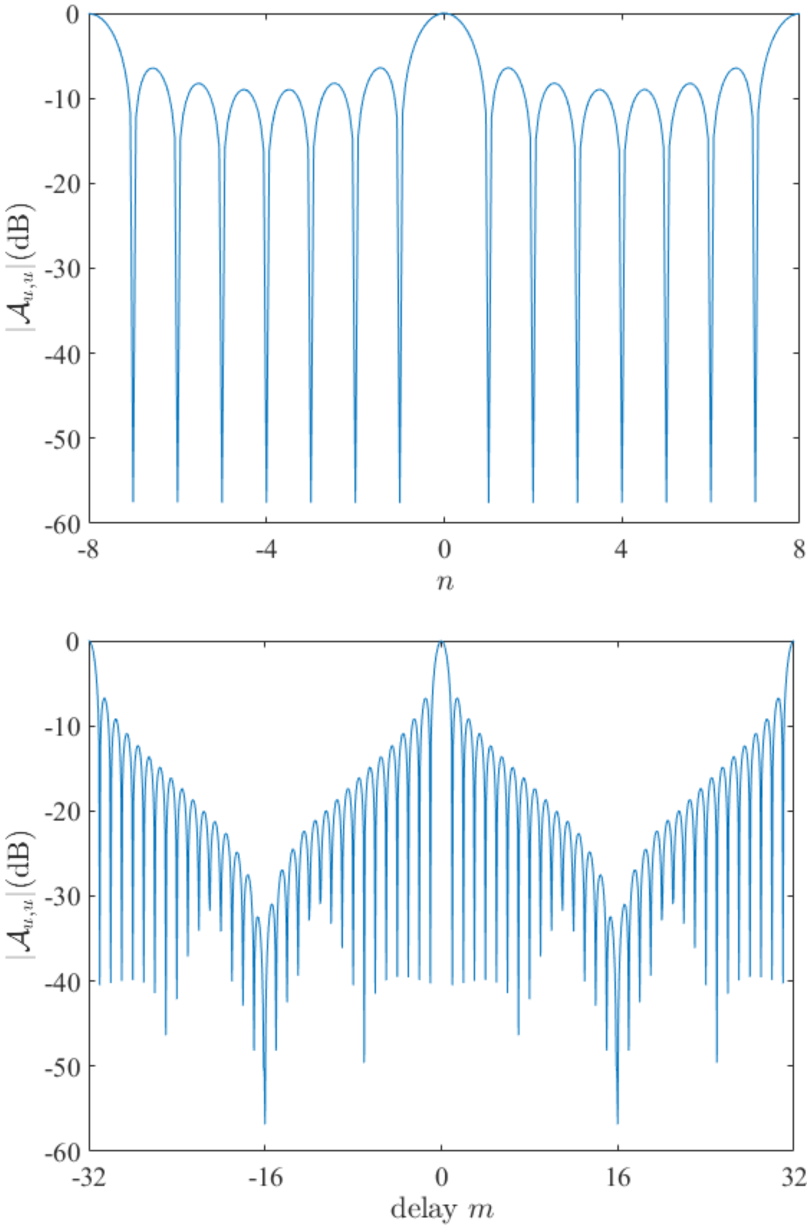
ODDM
DoF: MN

DDOP's Ambiguity Function

$$A_{u,u}\left(m\frac{T}{M}, n\frac{1}{NT}\right) = \delta(m)\delta(n), \forall|m| \leq M-1, |n| \leq N-1$$



$M = 32, N = 8, \rho = 0.1$



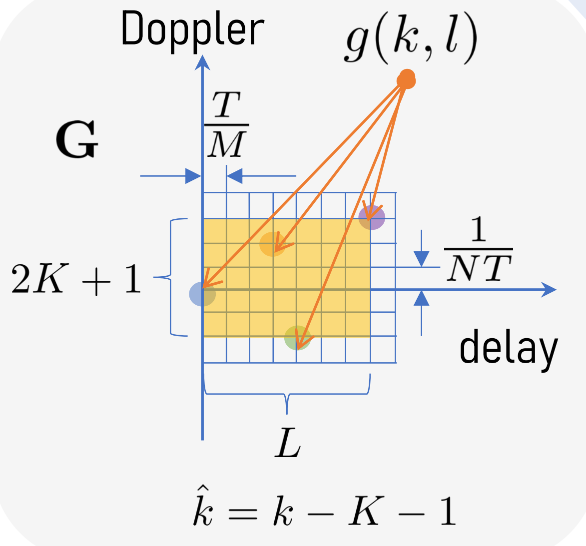
$m = 0$

$n = 0$

DD Domain Input-Output Relation

- Receive pulse (matched filter) : $u(t - m\frac{T}{M})e^{-j2\pi\frac{n}{NT}(t - m\frac{T}{M})}$
- Path's delay is integer multiples of $\frac{T}{M}$
- Path's Doppler is integer multiples of $\frac{1}{NT}$
- OFDM with integer timing/frequency offset
- See from the nth subcarrier of the mth symbol

$g(k,l)$: gain g corresponds to on-the-grid ISI (ICI) from the $[n-\hat{k}]_N$th subcarrier of the $[m-l]_M$th symbol



$$
\begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{M-1} \end{bmatrix}
=
\begin{bmatrix}
\mathbf{H}_0^0 & & & \mathbf{H}_{L-1}^0\mathbf{D} & \cdots & \cdots & \mathbf{H}_1^0\mathbf{D} \\
\vdots & \ddots & & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & & \ddots & \vdots \\
\mathbf{H}_{L-2}^{L-2} & \ddots & \ddots & \mathbf{H}_0^{L-2} & 0 & & \mathbf{H}_{L-1}^{L-2}\mathbf{D} \\
\mathbf{H}_{L-1}^{L-1} & \ddots & \ddots & \ddots & \mathbf{H}_0^{L-1} & & \\
 & \ddots & \ddots & \ddots & \ddots & \ddots & \\
 & & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & & \mathbf{H}_{L-1}^{M-1} & \ddots & \ddots & \ddots & \mathbf{H}_0^{M-1}
\end{bmatrix}
\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{M-1} \end{bmatrix}
+
\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_{M-1} \end{bmatrix}
$$

block-circulant-like structure

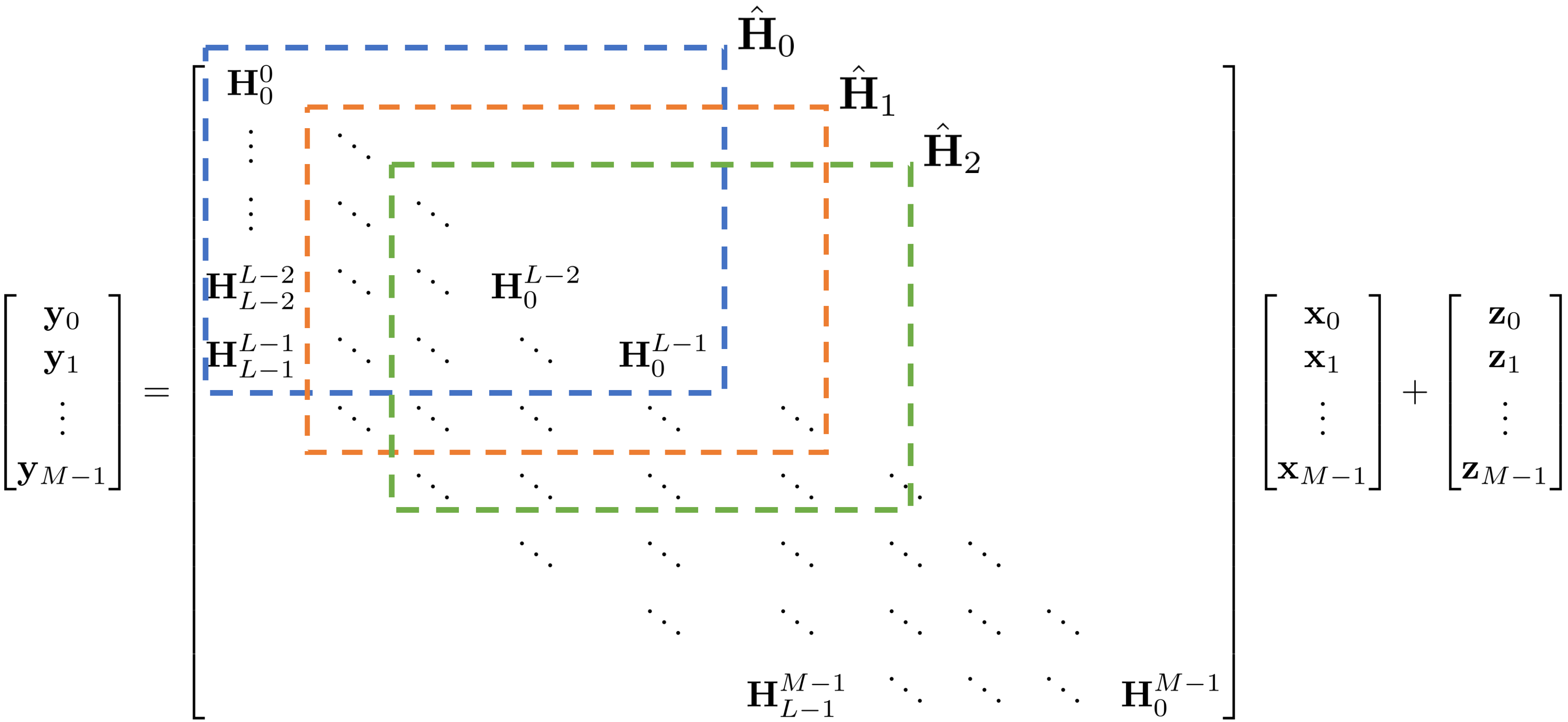
$$\mathbf{H}_l^m = \sum_{k=1}^{2K+1} g(k,l)e^{j2\pi\frac{\hat{k}(m-l)}{MN}}\mathbf{C}^{\hat{k}}$$

$$\mathbf{D} = \mathrm{diag}\left\{1, e^{-j\frac{2\pi}{N}}, \ldots, e^{-j\frac{2\pi(N-1)}{N}}\right\}$$

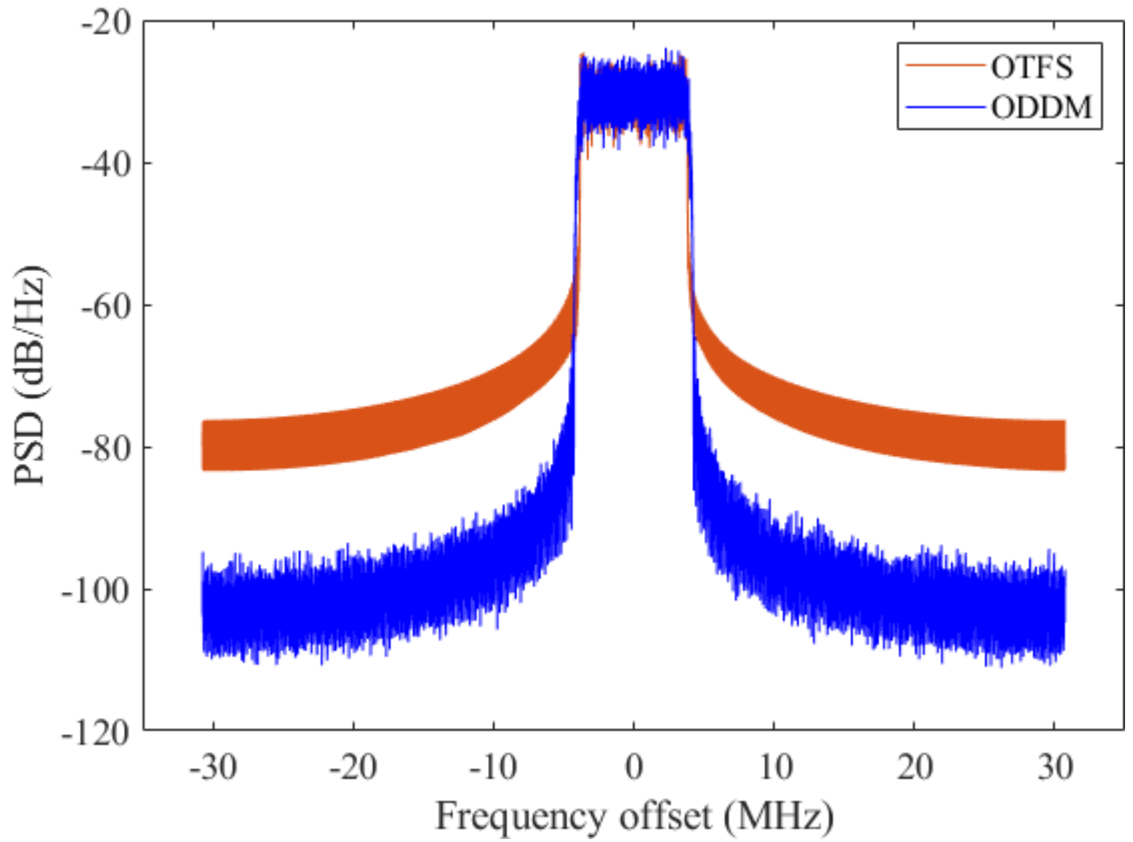
$\boldsymbol{C}$: $N \times N$ cyclic permutation matrix

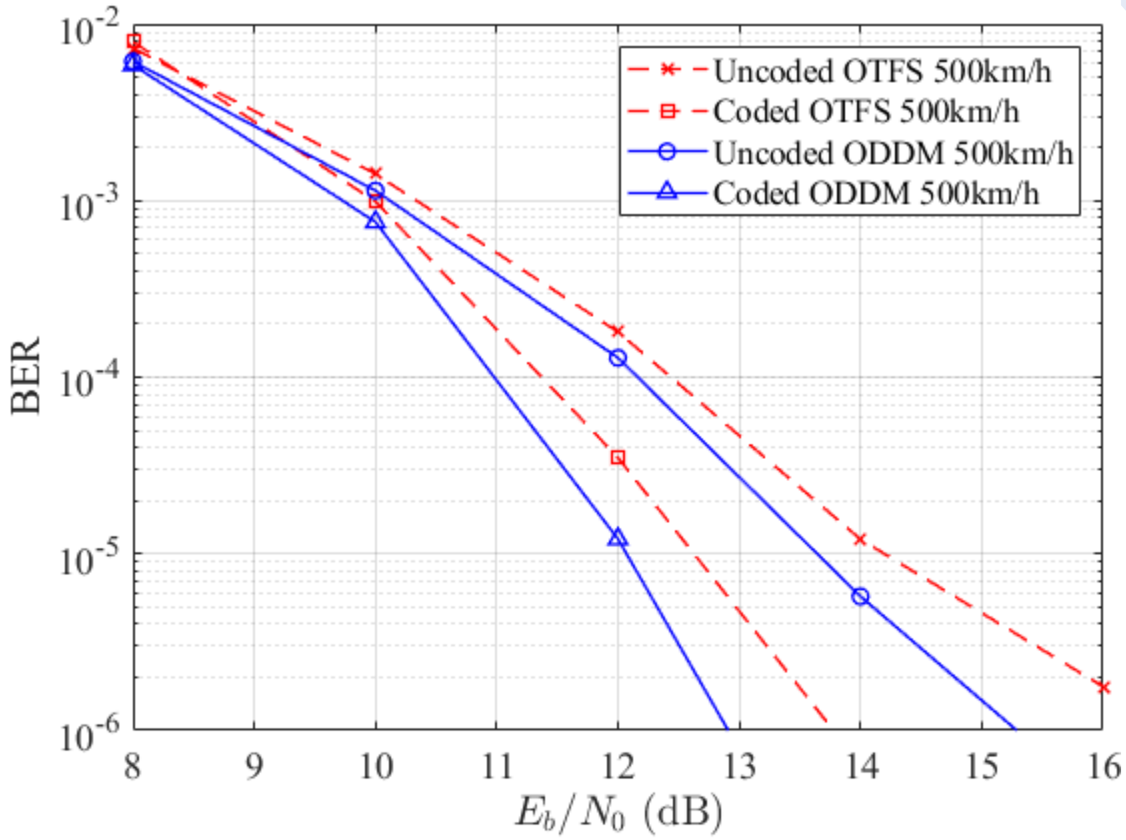
Detection Algorithms

- MAP: $O(X^{MN})$ MMSE: $O\left((MN)^3\right)$ MP: $O(MNPX)$
- For DDMC with ZP, Low-complexity MRC-SIC and MMSE-SIC : $O(MNL)$ and $O(MNL^3)$



Simulation Results





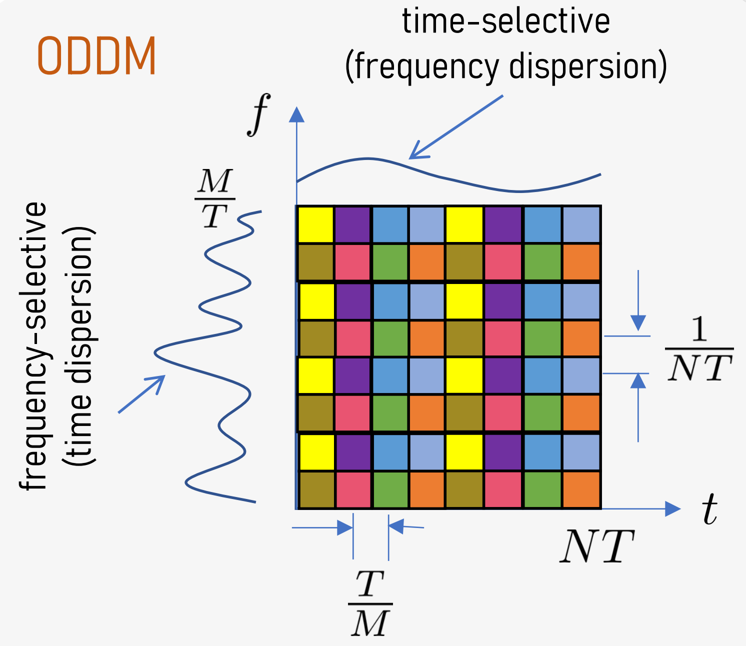
- $M = 512,\ N = 32,\ \frac{1}{T} = 15\text{kHz}, f_c = 5\text{GHz}$, EVA Channel
- $Q = 20$, roll-off factor = 0.1, 4-QAM, MP Equalization

Conclusion

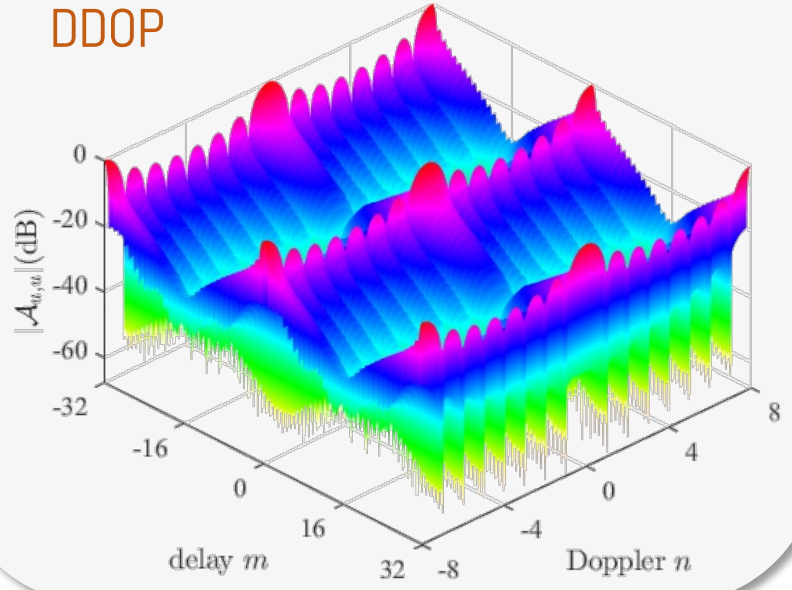
- A Novel Multi-Carrier Modulation Waveform
  - ✓ Embracing DD channel property
- DD Plane Orthogonal Pulse (DDOP)
- Potential for future
  - ✓ Reliable Communication for High Mobility
  - ✓ Integrated Sensing and Communication (ISAC)
- Many open issues . . .
- Demo code will be released soon at :

  https://www.omu.ac.jp/eng/ees-sic/oddm/

ODDM



DDOP



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- IEEE ICC 2023 Workshop on OTFS and DDMC for 6G, 28 May 2023, Rome, Italy
  - Submission deadline: January 20, 2023
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Thank you for your attention!