A Primer on Orthogonal Delay-Doppler Division Multiplexing (ODDM)

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SCAN ME

1. Introduction

Digital modulation scheme: Digital information ⇒ *Analog waveform* that matches the characteristics of the physical channel [1].

- *Real-valued* passband digital modulation waveform

$$x_{pb}(t) = \sum_i \sqrt{2}A_i \cos(2\pi(f_c + f_i(t))t + \theta_c + \theta_i)p_i(t), \tag{1}$$

  with carrier frequency f_c and phase θ_c, symbol index $i \in \mathbb{Z}$, amplitude A_i, phase θ_i, frequency $f_i(t)$, *finite* duration pulse $p_i(t)$.
- *Complex-valued* baseband waveform $x(t) = \sum_i x_i(t)$

$$x_i(t) = A_i e^{j\theta_i} e^{j2\pi f_i(t)t} p_i(t) \triangleq X[i]g_i(t), \tag{2}$$

  where each symbol has two components: an information-bearing *digital symbol* $X[i] = A_i e^{j\theta_i}$, and a finite-energy, continuous-time function $g_i(t) = e^{j2\pi f_i(t)t}p_i(t)$, referred to as *transmit (modulating) pulse*.

Design a digital modulation waveform: Determine a set of *orthogonal pulses*, also referred to as *basis functions*, to avoid inter-symbol interference (ISI).

2. Waveform Design Principles

LTI channels : Primary characteristic is the channel bandwidth $\mathbb{B}$.

- Single-Carrier (SC): Band-limited impulse based transmission with $f_i(t) = 0$, where each transmit pulse occupies the entire bandwidth and the pulses are multiplexed in time.
- Multi-Carrier (MC): (Truncated) Eigenfunctions based transmission with $f_i(t) = n\Delta F$, where each transmit pulse occupies a TF region to form a 2D multiplexing waveform

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=-N/2}^{N/2-1} X[m,n] \overbrace{e^{j2\pi n\Delta F(t-m\Delta T)} g(t - m\Delta T)}^{g_{m,n}(t)}, \tag{3}$$

  according to the symbol interval ΔT and the fundamental frequency ΔF. $e^{j2\pi n\Delta Ft}$ is often called *subcarrier* or *tone*, the transmit pulses $g_{m,n}(t)$ in (3) are truncated or pulse-shaped (PS) subcarriers, corresponding to *rectangular* or *non-rectangular* $g(t)$, respectively.
  - ✓ The channel bandwidth is matched by

$$B_x = (N-1) \times \Delta F + B_g \le \mathbb{B}, \tag{4}$$

    where B_x and B_g are the bandwidths of $x(t)$ and the prototype pulse $g(t)$, respectively.
  - ✓ TF domain MC (TFMC) waveform with a well-known design framework based on the Weyl-Heisenberg (WH) frame theory [2], where the pulses are considered as a WH set.
  - ✓ The WH frame theory indicates that there is no (bi)orthogonal WH set when the joint TF resolution $\Delta R = \Delta T \Delta F < 1$.
- TF occupancy of $g(t)$: TF area (TFA) given by $A_g = \alpha T_g \alpha B_g$, where αT_g and αB_g are the standard deviations of $g(t)$ and its Fourier transform $G(f)$, respectively.
  - ✓ The TF domain for pulse design is an *interdependent* 2D domain.
  - ✓ According to the uncertainty principle, the TFA obeys a lower bound $A_g \ge \frac{1}{4\pi}$ called the Gabor limit [3].
  - ✓ There is *no* 2D impulse that is "narrow" in time and frequency simultaneously (TF "narrow"), to be confined in a TF grid with small ΔT and ΔF.

LTV channels : Primary characteristics are the bandwidth $\mathbb{B}$ and the available time $\mathbb{T}$.

- The underspread LTV channels at best have a structured set of *approximate* eigenfunctions, which are even *channel-dependent* [4].
- During an appropriate $\mathbb{T}$, an LTV channel can be represented by a *deterministic* delay-Doppler (DD) spread function [5].
- Given a practical waveform bounded by bandwidth $\mathbb{B}$ and duration $\mathbb{T}$, the waveform received after receiving (or anti-aliasing) filtering can be sampled at an appropriate rate $B \le \mathbb{B}$ over a period of time $T \le \mathbb{T}$.
- The waveform may then be considered to have experienced an equivalent sampled DD (ESDD) channel associated with a delay resolution $\frac{1}{B}$ and a Doppler resolution $\frac{1}{T}$ [5].
- The waveform design for LTV channels may be replaced by that for ESDD channels, which have common characteristics of B and T or equivalently the DD resolution $(\frac{1}{B}, \frac{1}{T})$.
- *Physical units* of delay and Doppler are time and frequency, respectively. The DD domain modulation is naturally an MC modulation.
- Because of the joint TF resolution $\frac{1}{BT} \ll 1$ in practice, such DD domain MC (DDMC) modulation does not exist within the conventional TFMC waveform design framework.

3. ODDM Modulation

A novel MC modulation beyond the conventional TFMC waveform design framework.

- Given a design parameter T_0, we set $\Delta T = \frac{T_0}{M}$ and $\Delta F = \frac{1}{NT_0}$ in (3) for ODDM with $B = \frac{M}{T_0}$ and $T = NT_0$. The ODDM waveform without cyclic prefix can be represented as [6, 7]

$$\check{x}(t) = \sum_{m=0}^{M-1} \sum_{n=-N/2}^{N/2-1} X[m,n] e^{j2\pi \frac{n}{NT_0}\left(t - m\frac{T_0}{M}\right)} u\left(t - m\frac{T_0}{M}\right), \tag{5}$$

  where the DD domain orthogonal pulse (DDOP) $u(t)$ depicted in Figure 1 is a pulse-train defined as

$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0), \tag{6}$$

  whose elementary pulse $a(t)$ is a root Nyquist pulse parameterized by its Nyquist interval $\frac{T_0}{M}$ and duration $2Q\frac{T_0}{M}$.

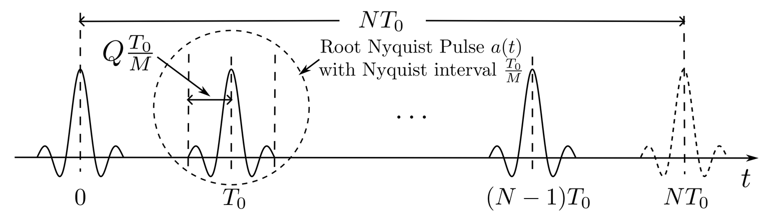

Figure 1: $u(t)$.

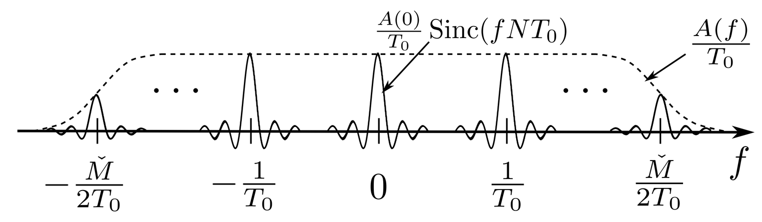

Figure 2: $U(f)$.

- When $2Q \ll M$, although $\frac{1}{BT} = \frac{1}{MN} \ll 1$, $u(t)$ satisfies the orthogonality property of

$$\mathcal{A}_{u,u}\left(m\frac{T_0}{M}, n\frac{1}{NT_0}\right) = \delta(m)\delta(n), \tag{7}$$

  for $|m| \le M-1$ and $|n| \le N-1$ [6], where $\mathcal{A}_{u,u}(\cdot)$ is the ambiguity function of $u(t)$ given by the inner product $\mathcal{A}_{u,u}(\tau,\nu) = \langle u(t), u(t-\tau)e^{j2\pi\nu(t-\tau)}\rangle$.
- Local or sufficient (bi)orthogonality: Only for MN pulses (WH subset) to circumvent the constraints imposed by the WH frame theory.
- Key ideas of DDOP : Instead of TF "narrow" pulse, use TF "wide" pulse (pulse train) to circumvent the uncertainty principle.
- The channel avaliable time is matached by

$$T_x = (M-1) \times \Delta T + T_g \le \mathbb{T}. \tag{8}$$

- (4) and (8) are also satisfied by setting $T_g \gg \Delta T$ and $B_g \gg \Delta F$, which require the prototype pulse to be TF "wide" and allows for staggering in both time and frequency domains.
- The DDOP has a wide bandwidth, long duration, and internal TF spaces among its elementary pulses.
- The DDOP may be considered as a *pseudo* 2D impulse, which behaves like the non-existent 2D impulse within a TF region of $(T_0, \frac{1}{T_0})$ w.r.t. the resolution of $(\frac{T_0}{M}, \frac{1}{NT_0})$.
- The DDOP-based ODDM achieves a pseudo-2D-impulse-based transmission over ESDD channels.

4. Implementation Methods

ODDM is a pulse-train-shaped (PTS) OFDM, and can be implemented as a PS-OFDM.

- PS-OFDM based implementation: For ODDM signal in (5), we have the mth symbol

$$\check{x}_m(t) = \overbrace{\left(\sum_{n=-N/2}^{N/2-1} X[m,n] e^{j2\pi \frac{n}{NT_0}\left(t - m\frac{T_0}{M}\right)}\right)}^{\dot{x}_m(t)} \times u\left(t - m\frac{T_0}{M}\right). \tag{9}$$

  - ✓ $\dot{x}_m(t) = \sum_{n=-N/2}^{N/2-1} X[m,n] e^{j2\pi \frac{n}{NT_0}\left(t - m\frac{T_0}{M}\right)}$ is strictly band-limited to $[-\frac{1}{2T_0}, \frac{1}{2T_0}]$ and can be sampled at a rate of $1/T_0$ to have $\dot{\mathbf{x}}_m = [\dot{x}_m[0], \cdots, \dot{x}_m[N-1]]^T$, which represents N samples sampled every T_0 within one period of NT_0.
  - ✓ $\dot{\mathbf{x}}_m$ is exactly the inverse discrete Fourier transform (IDFT) of $[X[m,0], \cdots, X[m,\frac{N}{2}-1], X[m,-\frac{N}{2}], \cdots, X[m,-1]]^T$.
  - ✓ We can pass *cyclic extension* (CE) of $\dot{\mathbf{x}}_m$ denoted by $\dot{\mathbf{x}}_m^{CE} = [\cdots, \dot{\mathbf{x}}_m^T, \dot{\mathbf{x}}_m^T, \dot{\mathbf{x}}_m^T \cdots]^T$ through ideal low-pass filter (LPF) with cut-off frequency $\frac{1}{2T_0}$ (a.k.a. interpolation filter) to obtain $\dot{x}_m(t)$ [8]. After that, the $u(t)$-based pulse-shaping is applied to $\dot{x}_m(t)$ to generate $\check{x}_m(t)$ in (9) as

$$\check{x}_m(t) = \overbrace{\left(\dot{\mathbf{x}}_m^{CE} * \mathrm{Sinc}\left(\frac{t}{T_0}\right)\right)}^{\dot{x}_m(t)} \times u\left(t - m\frac{T_0}{M}\right). \tag{10}$$

- Wideband filtered OFDM based approximate implementation: It is known that [6]

$$\check{x}_m(t) \approx \sum_{\dot{n}=0}^{N-1} \sum_{n=-N/2}^{N/2-1} X[m,n] e^{j2\pi\frac{n\dot{n}}{N}} a\left(t - \dot{n}T_0 - m\frac{T_0}{M}\right), \tag{11}$$

  where $\sum_{n=-N/2}^{N/2-1} X[m,n]e^{j2\pi\frac{n\dot{n}}{N}}, 0 \le \dot{n} \le N-1$ form $\dot{\mathbf{x}}_m$.
  - ✓ The mth ODDM symbol can be approximately generated by filtering $\dot{\mathbf{x}}_m$ with $a(t)$ as [6]

$$\check{x}_m(t) \approx \dot{\mathbf{x}}_m * a(t). \tag{12}$$

  - ✓ The fundamental frequency of the ODDM is $\Delta F = \frac{1}{NT_0}$. For a conventional filtered OFDM [9] with the same fundamental frequency, the cut-off frequency of the LPF or interpolation filter is around $\frac{1}{2T_0}$. Considering $a(t)$ has a cut-off frequency greater than $\frac{M}{2T_0}$, the approximate ODDM in (12) is a wideband filtered OFDM.
- *With appropriate parameter settings*, the wideband filtered OFDM based implementation can effectively generate the ODDM waveform.

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